Section: Turing Machines - Building Blocks

1. Given Turing Machines M1 and M2

Notation for

- Run M1
- Run M2

\[ M1 \rightarrow M2 \]

\[ z \in \text{any symbol in} \]

\[ z \rightarrow z, R \]

\[ z \rightarrow z, L \]
2. Given Turing Machines $M_1$ and $M_2$

$M_1$

$M_2$

$z$ represents any symbol in
$x$ is an element of

$M_1 \xrightarrow{x} M_2$

$z; z, L$

$x; x, R$
3. Given Turing Machines M1, M2, and M3

M1

M2

M3

x is an element of

y is any element except x from

z is any element from
More Notation for Simplifying Turing Machines

Suppose $\Gamma = \{a, b, c, B\}$

$z$ is any symbol in $\Gamma$

$x$ is a specific symbol from $\Gamma$

1. $s$ - start
2. $R$ - move right
3. $L$ - move left
4. $x$ - write $x$ (and don’t move)
5. $R_a$ - move right until you see an $a$
6. \( L_a \) - move left until you see an \( a \)

7. \( R_{\sim a} \) - move right until you see anything that is not an \( a \)

8. \( L_{\sim a} \) - move left until you see anything that is not an \( a \)

9. \( h \) - halt in a final state

10. \( \frac{a,b}{w} \rightarrow \)

   If the current symbol is a or b, let \( w \) represent the current symbol.
Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$. If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

input: bab, output: babb
input: ba, output: ba

What is the running time?

$\mathcal{O}(n)$
Example

Assume input string \( w \in \Sigma^+, \Sigma = \{a, b\}, |w| > 0 \)

For each \( a \) in the string, append a \( b \) to the end of the string.

input: \( a b b a b b \), output: \( a b b a b b b b \)

The tape head should finish pointing at the leftmost symbol of \( w \).
Turing’s Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

**Definition:** An *algorithm* for a function \( f : D \rightarrow R \) is a TM \( M \), which given input \( d \in D \), halts with answer \( f(d) \in R \).

**Example:** \( f(x + y) = x + y \), \( x \) and \( y \) unary numbers.

\[
\begin{align*}
\delta &= \{ \langle 1, + \rangle \} \\
\text{start with:} & \quad 111 + 1111 \\
\text{end with:} & \quad 1111111
\end{align*}
\]
Example: Copy a String, \( f(w) = w0w, \)
\( w \in \Sigma^*, \Sigma = \{a, b, c\} \)

Denoted by \( C \)

\[
\text{start with: } \quad \begin{array}{c}
B \\
\uparrow
\end{array}
\quad \begin{array}{c}
\text{Babac}
\end{array}
\quad \begin{array}{c}
B
\end{array}
\]

\[
\text{end with: } \quad \begin{array}{c}
B \\
\uparrow
\end{array}
\quad \begin{array}{c}
\text{Babac0abac}
\end{array}
\quad \begin{array}{c}
B
\end{array}
\]

Algorithm:

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol
Example: Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

start with: aaBba\_bca

↑

end with: aaBBbaca

↑
Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position
Example: Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

\[
\begin{align*}
\text{start with:} & \quad \text{babcaBba} \\
\text{end with:} & \quad \text{bacaBBba}
\end{align*}
\]

(similar to $S_R$)
Example: Add unary numbers
This time use shift.

Example: Multiply two unary numbers, f(x*y)=x*y, x and y unary numbers. Assume x,y>0.

start with: 1111*11
↑

end with: 11111111
↑