Section: Finite Automata

Deterministic Finite Accepter (or Automata)

A DFA = (Q, \Sigma, \delta, q_0, F)

where

Q is finite set of states
\Sigma is tape (input) alphabet
q_0 is initial state
F \subseteq Q is set of final states.
\delta: Q \times \Sigma \rightarrow Q
Example: DFA that accepts even binary numbers.

Transition Diagram:

```
M = (Q, Σ, δ, q₀, F) =
```

Tabular Format

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<thead>
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<tr>
<td>q₀</td>
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Example of a move: \( \delta(q₀, 1) = \)
Algorithm for DFA:

Start in start state with input on tape
q = current state
s = current symbol on tape
while (s != blank) do
    q = $\delta(q,s)$
    s = next symbol to the right on tape
if q $\in$ F then accept

Example of a trace: 11010
Pictorial Example of a trace for 100:

1) 

2) 

3) 

4)
Definition:

\[ \delta^*(q, \lambda) = q \]

\[ \delta^*(q, wa) = \delta(\delta^*(q, w), a) \]

Definition: The language accepted by a DFA \( M=(Q, \Sigma, \delta, q_0, F) \) is set of all strings on \( \Sigma \) accepted by \( M \). Formally, \( L(M) = \{ w \in \Sigma^* \mid \delta^*(q_0, w) \in F \} \)
Trap State

Example: $L(M) = \{a^n b a^n | n > 0 \}$
Example:

\[ L = \{ w \in \Sigma^* \mid w \text{ has an even number of a's and an even number of b's} \} \]
Example: DFA that accepts even binary numbers that have an even number of 1’s.

Two solutions:
Definition A language $L$ is regular iff there exists DFA $M$ s.t. $L = L(M)$. 
Chapter 2.2

Nondeterministic Finite Automata (or Accepter)

Definition

An NFA = (Q, Σ, δ, q₀, F)

where

Q is finite set of states
Σ is tape (input) alphabet
q₀ is initial state
F ⊆ Q is set of final states.

δ: Q × (Σ ∪ {λ}) → 2^Q
Example

Note: In this example $\delta(q_0, a) = \{q_1, q_2\}$

$L = \{aabb^n | n \geq 0 \}$
Example

$$L = \{(ab)^n \mid n > 0\} \cup \{a^n b \mid n > 0\}$$
Definition $q_j \in \delta^*(q_i, w)$ if and only if there is a walk from $q_i$ to $q_j$ labeled $w$.

Example From previous example:

\[
\begin{align*}
\delta^*(q_0, ab) &= \{q_1, q_4, q_6, 3\} \\
\delta^*(q_0, aba) &= \{q_3, q_5, 2\}
\end{align*}
\]

Definition: For an NFA $M$, $L(M) = \{w \in \Sigma^* \mid \delta^*(q_0, w) \cap F \neq \emptyset\}$
2.3 NFA vs. DFA: Which is more powerful? **Neither**

Example:
Theorem Given an NFA $M_N=(Q_N, \Sigma, \delta_N, q_0, F_N)$, then there exists a DFA $M_D=(Q_D, \Sigma, \delta_D, q_0, F_D)$ such that $L(M_N) = L(M_D)$.

Proof:

We need to define $M_D$ based on $M_N$.

$Q_D = 2^{Q_N}$

$F_D = \{ Q \subseteq Q_D \mid \exists q_i \in Q_N \text{ with } q_i \in F_N \}$

$\delta_D : Q_D \times \Sigma \rightarrow Q_D$
Algorithm to construct $M_D$

1. start state is $\{q_0\} \cup \text{closure}(q_0)$

2. While can add an edge
   
   (a) Choose a state $A=\{q_i, q_j, \ldots q_k\}$ with missing edge for $a \in \Sigma$
   
   (b) Compute $B = \delta^*(q_i, a) \cup \delta^*(q_j, a) \cup \ldots \cup \delta^*(q_k, a)$
   
   (c) Add state $B$ if it doesn’t exist
   
   (d) add edge from $A$ to $B$ with label $a$

3. Identify final states

4. if $\lambda \in L(M_N)$ then make the start state final.
Example:
Properties and Proving - Problem 1

Consider the property
Replace_one_a_with_b or R1awb for short. If L is a regular, prove R1awb(L) is regular.

The property R1awb applied to a language L replaces one a in each string with a b. If a string does not have an a, then the string is not in R1awb(L).

Example 1: Consider L=\{aaab, bbaa\}
R1awb(L)=\{bab, aab, aabb, baba, bbaa\}

Example 2: Consider \( \Sigma = \{a, b\} \), L = \{\( w \in \Sigma^* \mid w \text{ has an even number of a’s and an even number of b’s} \}\)
R1awb(L)=\{w \in \Sigma^* \mid w \text{ has an odd number of a’s and an odd number of b’s} \}

Proof:
Properties and Proving - Problem 2

Consider the property Truncate_all_preceeding_b’s or TruncPreb for short. If L is a regular, prove TruncPreb(L) is regular.

The property TruncPreb applied to a language L removes all preceeding b’s in each string. If a string does not have an preceeding b, then the string is the same in TruncPreb(L).

Example 1: Consider \( L = \{aaab, bbab\} \)

\[ \text{TruncPreb}(L) = \]

Example 2: Consider \( L = \{(bba)^n \mid n > 0\} \)

\[ \text{TruncPreb}(L) = \]

Proof:
Minimizing Number of states in DFA

Why?

Algorithm

- Identify states that are indistinguishable
  These states form a new state

Definition Two states $p$ and $q$ are indistinguishable if for all $w \in \Sigma^*$

\[
\delta^*(q, w) \in F \Rightarrow \delta^*(p, w) \in F
\]
\[
\delta^*(p, w) \notin F \Rightarrow \delta^*(q, w) \notin F
\]

Definition Two states $p$ and $q$ are distinguishable if $\exists w \in \Sigma^*$ s.t.

\[
\delta^*(q, w) \in F \Rightarrow \delta^*(p, w) \notin F \text{ OR }
\]
\[
\delta^*(q, w) \notin F \Rightarrow \delta^*(p, w) \in F
\]
Example:
Example: