Section: Pushdown Automata

Ch. 7 - Pushdown Automata

A DFA $= (Q, \Sigma, \delta, q_0, F)$

input tape

| a | a | b | b | a | b |

tape head

head moves

current state

0 1
Modify DFA by adding a stack. New machine is called Pushdown Automata (PDA).
Definition: Nondeterministic PDA (NPDA) is defined by

\[ M = (Q, \Sigma, \Gamma, \delta, q_0, z, F) \]

where
- \( Q \) is finite set of states
- \( \Sigma \) is tape (input) alphabet
- \( \Gamma \) is stack alphabet
- \( q_0 \) is initial state
- \( z \) - start stack symbol, \( z \in \Gamma \)
- \( F \subseteq Q \) is set of final states.

\[ \delta: Q \times (\Sigma \cup \{\lambda\}) \times \Gamma \rightarrow \text{finite subsets of } Q \times \Gamma^* \]
Example of transitions

\[ \delta(q_1, a, b) = \{(q_3, b), (q_4, ab), (q_6, \lambda)\} \]

The diagram for the above transitions is:
Instantaneous Description:

(q, w, u)

Description of a Move:

(q_1, aw, bx) \vdash (q_2, w, yx)

iff

(q_2, y) \in S(q_1, a_1 b)

Definition Let M=\( (Q, \Sigma, \Gamma, \delta, q_0, z, F) \) be a NPDA. \( L(M) = \{ w \in \Sigma^* \mid (q_0, w, z) \vdash^* (p, \lambda, u), \ p \in F, \ u \in \Gamma^* \} \). The NPDA accepts all strings that start in \( q_0 \) and end in a final state.
Example: \( L = \{ a^n b^n | n \geq 0 \} \), \( \Sigma = \{ a, b \} \), \( \Gamma = \{ z, a \} \)
Another Definition for Language Acceptance

NPDA $M$ accepts $L(M)$ by empty stack:

$$L(M) = \{ w \in \Sigma^* | (q_0, w, z) \vdash^* (p, \lambda, \lambda) \}$$
Example: \( L = \{a^n b^m c^{n+m} | n, m > 0 \} \),
\( \Sigma = \{a, b, c\} \), \( \Gamma = \{0, z\} \)
Examples for you to try on your own: (solutions are at the end of the handout).

- \( L = \{ a^n b^m \mid m > n, m, n > 0 \} \), \( \Sigma = \{ a, b \} \), \( \Gamma = \{ z, a \} \)
- \( L = \{ a^n b^{n+m} c^m \mid n, m > 0 \} \), \( \Sigma = \{ a, b, c \} \)
- \( L = \{ a^n b^{2n} \mid n > 0 \} \), \( \Sigma = \{ a, b \} \)
Definition: A PDA $M=(Q, \Sigma, \Gamma, \delta, q_0, z, F)$ is deterministic if for every $q \in Q$, $a \in \Sigma \cup \{\lambda\}$, $b \in \Gamma$

1. $\delta(q, a, b)$ contains at most 1 element
2. if $\delta(q, \lambda, b) \neq \emptyset$ then $\delta(q, c, b) = \emptyset$ for all $c \in \Sigma$

Definition: $L$ is DCFL iff $\exists$ DPDA $M$ s.t. $L = L(M)$.
Examples:

1. Previous pda for \( \{a^n b^n | n \geq 0\} \) is deterministic?

2. Previous pda for 
\( \{a^n b^m c^{n+m} | n, m > 0\} \) is deterministic?

3. Previous pda for 
\( \{w w^R | w \in \Sigma^+\}, \Sigma = \{a, b\} \) is deterministic?
Example: \( L = \{ a^n b^m | m > n, m, n > 0 \} \), 
\( \Sigma = \{ a, b \} \), \( \Gamma = \{ z, a \} \)

Example: \( L = \{ a^n b^{n+m} c^m | n, m > 0 \} \), 
\( \Sigma = \{ a, b, c \} \)

Example: \( L = \{ a^n b^{2n} | n > 0 \} \), \( \Sigma = \{ a, b \} \)