Combining Turing Machines

We will define notation that will make it easier to look at more complicated Turing machines

1. Given Turing Machines M1 and M2
   Notation for
   - Run M1
   - Run M2

   ![Diagram 1](image1)

   z represents any symbol in \( \Gamma \)

2. Given Turing Machines M1 and M2
   Notation for
   - Run M1
   - If x is current symbol
     - then Run M2

   ![Diagram 2](image2)

   x is an element of \( M1, M2 \)
3. Given Turing Machines M1, M2, and M3

Notation for

- Run M1
- If x is current symbol
  - then Run M2
  - else Run M3

More Notation for Simplifying Turing Machines

Suppose \( \Gamma = \{a, b, c, B\} \)

- z is any symbol in \( \Gamma \)
- x is a specific symbol from \( \Gamma \)

1. s - start
2. R - move right
3. L - move left

4. x - write x (and don’t move)

5. R_a - move right until you see an a

6. L_a - move left until you see an a

7. R_{\sim a} - move right until you see anything that is not an a

8. L_{\sim a} - move left until you see anything that is not an a

9. h - halt in a final state

10. \[ \Rightarrow_w \]

   If the current symbol is a or b, let w represent the current symbol.
Example

Assume input string \( w \in \Sigma^+ \), \( \Sigma = \{a, b\} \).

If \(|w|\) is odd, then write a \( b \) at the end of the string. The tape head should finish pointing at the leftmost symbol of \( w \).

input: bab, output: babb

input: ba, output: ba

What is the running time?
Example

Assume input string \( w \in \Sigma^+, \Sigma = \{a, b\}, \ |w| > 0 \)

For each \( a \) in the string, append a \( b \) to the end of the string.

input: \( abbabb \), output: \( abbabbb \)

The tape head should finish pointing at the leftmost symbol of \( w \).

Turing’s Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An algorithm for a function \( f: \mathbb{D} \rightarrow \mathbb{R} \) is a TM \( M \), which given input \( d \in \mathbb{D} \), halts with answer \( f(d) \in \mathbb{R} \).

Example: \( f(x + y) = x + y \), \( x \) and \( y \) unary numbers.

\[
\begin{array}{l}
\text{start with: } 111+1111 \\
\uparrow \\
\text{end with: } 1111111 \\
\uparrow 
\end{array}
\]
**Example:** Copy a String, \( f(w) = w0w, \ w \in \Sigma^*, \ \Sigma = \{a, b, c\} \)

Denoted by \( C \)

```
start with:    abac
    ↑
end with:    abac0abac
    ↑
```

**Algorithm:**

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol
**Example:** Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

- Start with: $aaBbabca$
- End with: $aaBBbaca$

**Algorithm:**

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position
Example: Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

start with: babcaBba

end with: bacaBBba

(similar to $S_R$)
Example: Add unary numbers
This time use shift.

Example: Multiply two unary numbers, \( f(x*y) = x*y \), \( x \) and \( y \) unary numbers. Assume \( x,y > 0 \).

\[
\begin{align*}
\text{start with:} & \quad 1111*11 \\
& \quad \uparrow \\
\text{end with:} & \quad 11111111 \\
& \quad \uparrow 
\end{align*}
\]