Section: Turing Machines - Building Blocks

1. Given Turing Machines M1 and M2

Notation for

- Run M1
- Run M2

\[ M_2 \rightarrow M_1 \rightarrow M_2 \]

\[ S \quad H \quad z; z, R \quad z; z, L \quad S' \quad H' \]

\( z \) represents any symbol in
2. Given Turing Machines M1 and M2

\[ \rightarrow M1 \xrightarrow{x} M2 \]

\[ z \text{ represents any symbol in} \]

\[ x \text{ is an element of} \]
3. Given Turing Machines M1, M2, and M3

- M1
- M2
- M3

x is an element of
y is any element except x from
z is any element from
More Notation for Simplifying Turing Machines

Suppose $\Gamma=\{a,b,c,B\}$

- $z$ is any symbol in $\Gamma$
- $x$ is a specific symbol from $\Gamma$

1. $s$ - start
2. $R$ - move right
3. $L$ - move left
4. $x$ - write $x$ (and don’t move)
5. $R_a$ - move right until you see an $a$
6. $L_a$ - move left until you see an $a$

7. $R_{\neg a}$ - move right until you see anything that is not an $a$

8. $L_{\neg a}$ - move left until you see anything that is not an $a$

9. $h$ - halt in a final state

10. $\{a,b\} \rightarrow w$

If the current symbol is $a$ or $b$, let $w$ represent the current symbol.
Example

Assume input string \( w \in \Sigma^+ \), \( \Sigma = \{a, b\} \). If \( |w| \) is odd, then write a \( b \) at the end of the string. The tape head should finish pointing at the leftmost symbol of \( w \).

input: \( bab \), output: \( babb \)
input: \( ba \), output: \( ba \)

What is the running time?
Example

Assume input string $w \in \Sigma^+, \Sigma = \{a, b\}, |w| > 0$

For each $a$ in the string, append a $b$ to the end of the string.

input: $abbabb$, output: $abbabbbb$

The tape head should finish pointing at the leftmost symbol of $w$. 
Turing’s Thesis: Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An *algorithm* for a function $f: \mathbb{D} \to \mathbb{R}$ is a TM $M$, which given input $d \in \mathbb{D}$, halts with answer $f(d) \in \mathbb{R}$.

Example: $f(x + y) = x + y$, $x$ and $y$ unary numbers.

start with: $\begin{array}{c}111+1111 \\ \uparrow \end{array}$

end with: $\begin{array}{c}1111111 \\ \uparrow \end{array}$
Example: Copy a String, \( f(w) = w0w \), \( w \in \Sigma^* \), \( \Sigma = \{a, b, c\} \)

Denoted by \( C \)

\[
\begin{align*}
\text{start with:} & \quad \text{abac} \\
& \uparrow \\
\text{end with:} & \quad \text{abac0abac} \\
& \uparrow
\end{align*}
\]

Algorithm:

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol
Example: Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

start with: aaBbabc

↑

end with: aaBBbaca

↑
Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position
Example: Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

start with: \[ \text{babcaBba} \]

$\uparrow$

end with: \[ \text{bacaBBba} \]

$\uparrow$

(similar to $S_R$)
Example: Add unary numbers
This time use shift.

Example: Multiply two unary numbers, \( f(x \times y) = x \times y \), \( x \) and \( y \) unary numbers. Assume \( x, y > 0 \).

\[
\text{start with: } \quad 1111 \times 11 \\
\quad \uparrow
\]

\[
\text{end with: } \quad 11111111 \\
\quad \uparrow
\]