Review

Regular Languages

- FA, RG, RE
- recognize

Context Free Languages

- PDA, CFG
- recognize

DFA:

Turing Machine:
Turing Machine (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms

Definition of TM

- Storage
  - tape
- actions
  - write symbol
  - read symbol
  - move left (L) or right (R)
- computation
  - initial configuration
    * start state
    * tape head on leftmost tape square
    * input string followed by blanks
  - processing computation
    * move tape head left or right
    * read from and write to tape
  - computation halts
    * final state

Formal Definition of TM

A TM $M$ is defined by $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$ where

- $Q$ is finite set of states
- $\Sigma$ is input alphabet
- $\Gamma$ is tape alphabet
- $B\in\Gamma$ is blank
- $q_0$ is start state
- $F$ is set of final states
- $\delta$ is transition function

$\delta(q,a) = (p,b,R)$ means “if in state $q$ with the tape head pointing to an 'a', then move into state $p$, write a 'b' on the tape and move to the right”.
TM as Language recognizer

**Definition:** Configuration is denoted by \( \vdash \).

If \( \delta(q,a) = (p,b,R) \) then a move is denoted

\[
\text{abaqabba} \vdash \text{abapbbba}
\]

**Definition:** Let \( M \) be a TM, \( M=(Q,\Sigma,\Gamma,\delta,q_0,B,F) \). \( L(M) = \{ w \in \Sigma^* | q_0w \vdash x_1q_fx_2 \text{ for some } q_f \in F, \ x_1, x_2 \in \Gamma^* \} \)

TM as language acceptor

\( M \) is a TM, \( w \) is in \( \Sigma^* \),

- if \( w \in L(M) \) then \( M \) halts in final state
- if \( w \not\in L(M) \) then either
  - \( M \) halts in non-final state
  - \( M \) doesn’t halt

TM as a transducer

TM can implement a function: \( f(w) = w' \)

\[
\begin{align*}
\text{start with:} & \quad w \\
& \quad \uparrow \\
\text{end with:} & \quad w' \\
& \quad \uparrow
\end{align*}
\]
**Definition:** A function with domain $D$ is *Turing-computable or computable* if there exists TM $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$ such that

$$q_0w \xrightarrow{*} q_f f(w)$$

$q_f \in F$, for all $w \in D$.

**Example**

$\Sigma = \{a, b\}$

Replace every second 'a' by a 'b' if string is even length.

- Algorithm
Example:

\[ L = \{a^n b^n c^n | n \geq 1\} \]

Is the following TM Correct?

Example:

\[ f(x) = 2x \]

\( x \) is a unary number

\[
\begin{align*}
\text{start with:} & \quad 111 \\
\uparrow & \\
\text{end with:} & \quad 111111 \\
\uparrow & \\
\end{align*}
\]

Is the following TM correct?
Example:

$L = \{ w w \mid w \in \Sigma^+ \}$, $\Sigma = \{a, b\}$