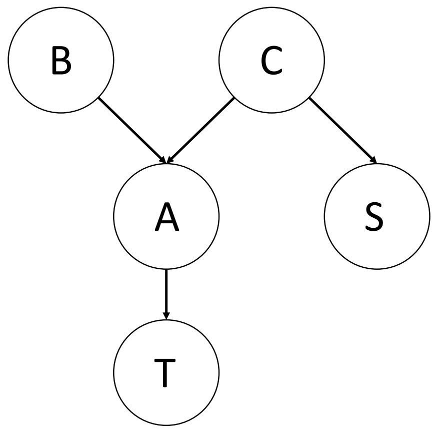
COMPSCI 370	**Instructor:** Ronald Parr
Homework 4	**Due:** Thursday, April 13, 2023

Updated at 10:00 AM on 4/6 to fix typos in question 4.

Our Bayes net questions will use the following network. Note that all variables are binary:



We motivate this structure with the following scenario: You have a crude burglar alarm installed at your house (A). Two things can cause the alarm to go off, a burglar (B) or vibration from construction in your neighborhood (C). If your alarm is ringing, you may get a text from your neighbor (T). If there is construction in your neighborhood, there might be a post about the construction on social media (S).

This network has the following CPTs. Note that the CPTs have probabilities that are convenient, not necessarily realistic:

$$P(b) = 1/8$$

$$P(c) = 1/4$$

B	C	$P(a\|BC)$
$\overline{b}$	$\overline{c}$	1/8
$\overline{b}$	c	3/8
b	$\overline{c}$	7/8
b	c	1

C	$P(s\|C)$
$\bar{c}$	1/8
c	3/4

A	$P(t\|A)$
$\bar{a}$	1/8
a	3/4

For all of the following questions, use variable elimination to compute the desired quantities. Show your work and any intermediate results computed explicitly.

1 Bayes Nets I (10 points)

Compute the marginal probability: $P(t)$.

2 Bayes Nets II (10 points)

Compute $P(b|t)$, the probability of burglary given that you got a text. (You can do this by computing $P(bt)$). Note that you should expect this to be higher than the prior probability of burglary, $P(b) = 1/8$.

3 Bayes Nets III (10 points)

Compute $P(st)$, the probability of a social media report about construction, and a text from your neighbor about your alarm.

4 Bayes Nets IV (10 points)

Compute $P(b|st)$. You can do this by computing $P(bst)$. This is the probability of burglary given *both* a social media report and a text from your neighbor. Comparing this with $P(b|t)$, you should expect a lower probability for this question. Why? The report about construction partly *explains away* the burglary as a cause for the text.

5 HMMs (30 points)

Consider a hidden Markov model with two states, x and y, two observations a and b, and the following somewhat unusual parameters. We use the random variable S_t for the state at time t:

- $P(S_{t+1} = x|S_t = x) = P(S_{t+1} = y|S_t = y) = 1$
- $P(a|S_t = x) = P(b|S_t = x) = 0.5$

- $P(b|S_t = y) = 1.0$

These parameters were chosen to help you realize some things about HMMs as you work through the basic algorithms. Assume that at time 0, the distribution over states is: $P(S_0 = x) = 0.6$, and that there are no observations at time 0. At time 1, the observation is b. At time 2, the observation is a.

a) Show the work to run the forward algorithm (AKA monitoring or tracking) and compute the distribution over states at time steps 1 and 2, i.e., $P(S_1|O_1 = b)$ and $P(S_2|O_1 = b, O_2 = a)$.

b) Show the work to compute the smoothed distribution over states at time step 1, i.e., $P(S_1|O_1 = b, O_2 = a)$, where O_t is the observation at time t. (Hint: You should see a big difference between the forward probabilities at time 1 and the smoothed probabilities.)

c) Show the work to compute the Viterbi path given the same initial distribution at time 0, and same observatins, $O_1 = b$, then extend your calculation to include $O_2 = a$. Note that final your answers should be a sequence of 2 states up to $O_1 = b$, and then a sequence of 3 states when you extend your answer. You should observe something interesting about how the optimal path changes when you get the observation at time 2.