L7: Runtime Efficiency

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CompSci 201: Spring 2024

2/5/24

Logistics, Coming up

- Today 2/6
  - Project 1 NBody due today
  - Project 2 Markov releasing tomorrow (due in 2 weeks)
- Wednesday 2/8
  - APT 3 due
- Next Monday 2/13
  - Midterm Exam 1
    - Covers everything through THIS week, up to and including asymptotic analysis / Big O
    - Example/Practice exams available this evening

Midterm Exams

See details on course website assignments and grading page

- 60 minutes, in-class exam
- Multiple choice + short answer
- Reason about algorithms, data structures, code.
- Can bring 1 double sided reference sheet (8.5x11 in), write/type whatever notes you like.
- No electronic devices out during exam

Exam Grades and Missing Exams

- Three midterm exams scheduled: (E1, E2, E3)
- Final exam has 3 corresponding parts: (F1, F2, F3)
    - Worth 11% of grade

- Overall exam grade has four exam parts
    - Part 1, Part 2, Part 3, Final: 11% each
    - Part i grade: max(Ei, Fi)

- Meaning the final exam serves in part:
    - As a makeup, if you need to miss a midterm, and/or
    - As an opportunity to demonstrate more learning, if you're unhappy with your midterm score.

Midterm 1 Material/Concepts

- Lectures 1-8
  - WOTO answers being added to early slides tonight
- Discussion 1-4
  - Solutions to documents on schedule since D1
- Project 0, Project 1
- APT 1-3, *required* problems
  - Optional/challenge not expected, but great practice

Midterm 1 Material/Concepts

- Java
  - Methods (return types, parameters)
  - Classes/Objects (instance variables, constructors)
  - Primitive types (e.g., int, double,…) vs. primitive types (e.g., Integer, Double, String, [any class])
  - Immutability (e.g. of primitive types and Strings)
  - Static vs. non-static
  - Overriding methods (e.g., .equals, .hashcode, .toString)

- List/Set/Map ADTs
  - Methods / APIs
  - ArrayList impl. of List
  - HashMap impl. of Map (hash tables) using ArrayList for buckets
    - .equals/.hashcode contract
  - Efficiency of implementation's methods, e.g., of .contains

- Asymptotic Analysis / Runtime Efficiency
  - Big O notation
  - Analyze runtime of code snippets *with respect to parameter size*

Wrapping up Maps

https://www.buzzfeed.com/maitlandquitmeyer/gift-wrap-ideas-that-are-almost-too-pretty-to-tear-apart?utm_term=.dydzwPbor&sub=4413758_10120650

HashMap methods at a high level

Always start by getting the **`hash`** =

`Math.abs(key.hashCode()) % list.size()`

Absolute value and % (remainder when dividing by) list size ensures valid index

- `put(key, value)`
  - Add <`key,value`> pair to "bucket" (list) at index **`hash`** if existing pair with same key
  - Otherwise replace existing pair with given `value`
- `get(key)`
  - Return value paired with **`key`** in bucket at index **`hash`** (or return **`null`** if no such pair)
- `containsKey(key)`
  - Check if key exists in bucket at index **`hash`**

0	
1	<"hi", 5> <"fain", 104>
2	
3	
4	<"ok", 3>
5	
6	
7	

Revisiting Hashing Efficiency

- Runtime of **`get()`**, **`put()`**, and **`containsKey()`**

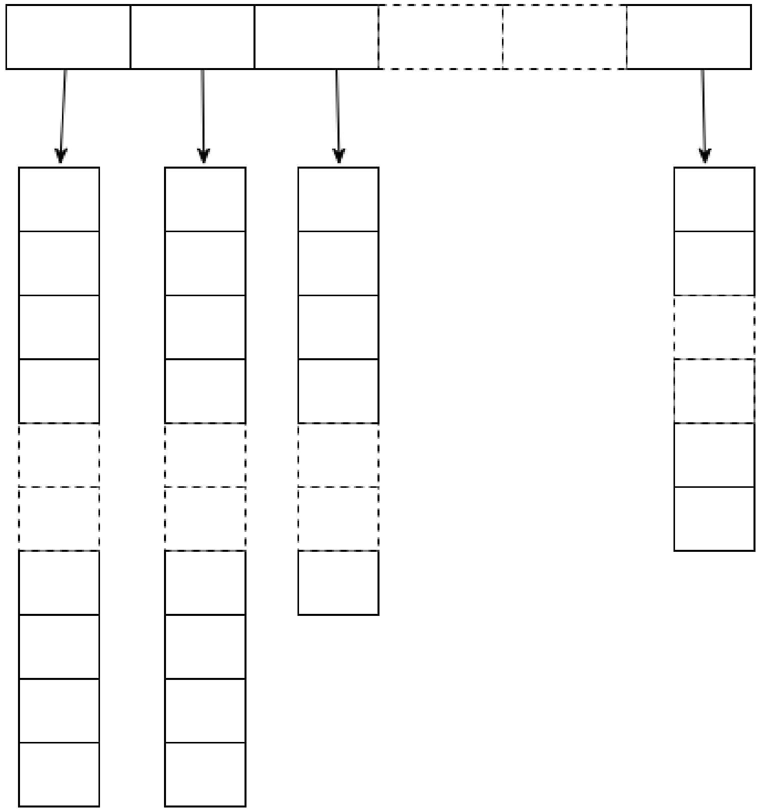
  = Time to get the hash

  + Time to search "bucket" by calling .equals() on everything in the bucket

Constant, does not depend on number of pairs in Map

Depends on number of pairs per bucket

⇒ HashMaps are faster with more buckets



Simple uniform hashing assumption (SUHA)

- Suppose we hash N pairs to M buckets.

- *Simple uniform hashing assumption*:

  Any element (i.e., key for HashMap, value for HashSet) is **equally likely** to hash into any bucket, **independently** of where any other element hashes to. [CLRS]

- Probability any two unequal elements hash into the same bucket: 1/M
  - Spread of pairs to buckets **looks random** (but is not).
  - Ways to design such hash functions, not today
  - We make this assumption to analyze efficiency in theory, can verify runtime performance in practice

Implications of SUHA

- Expected number of pairs per bucket under SUHA? N/M [N pairs, M buckets].

- Stronger statements are true: Very high probability that any bucket has approximately N/M pairs.

Constant, does not depend on N or M.

- Runtime implication?
  - Time to get the hash
  - Time to search over the "bucket" at hash index
    - Calling .equals() on everything in the bucket

Expect ~ N/M pairs to search

Memory/Runtime Tradeoff

- N pairs, M buckets, assuming SUHA / good `hashCode()`

- Case 1: N >> M – too many pairs in too few buckets
  - Runtime inefficient

- Case 2: M >> N – too many buckets, not many pairs
  - Runtime efficient, NOT memory efficient

- Case 3: M slightly larger than N – sweet spot
  - Runtime efficient, memory usage slightly more than an array/ArrayList

Load Factor and HashMap Growth

- N pairs, M buckets

- *Load factor* = maximum N/M ratio allowed
    - Java default is 0.75

- Whenever N/M exceeds the load factor?
    - Create a new larger table, rehash/copy everything
    - Double the size, just like ArrayList!
        - Geometric growth pattern for amortized efficiency
    - Called *resizing*

Hash table resizing

```
[jshell> Math.abs("cs".hashCode()) % 4
$15 ==> 0
[jshell> Math.abs("hi".hashCode()) % 4
$16 ==> 1
[jshell> Math.abs("ok".hashCode()) % 4
$17 ==> 0
```

```
[jshell> Math.abs("cs".hashCode()) % 8
$19 ==> 0
[jshell> Math.abs("hi".hashCode()) % 8
$20 ==> 1
[jshell> Math.abs("ok".hashCode()) % 8
$21 ==> 4
```

0	<“cs”, 201> <“ok”, 3>
1	<“hi”, 5>
2	
3	

Resizing

0	<“cs”, 201>
1	<“hi”, 5>
2	
3	
4	<“ok”, 3>
5	
6	
7	

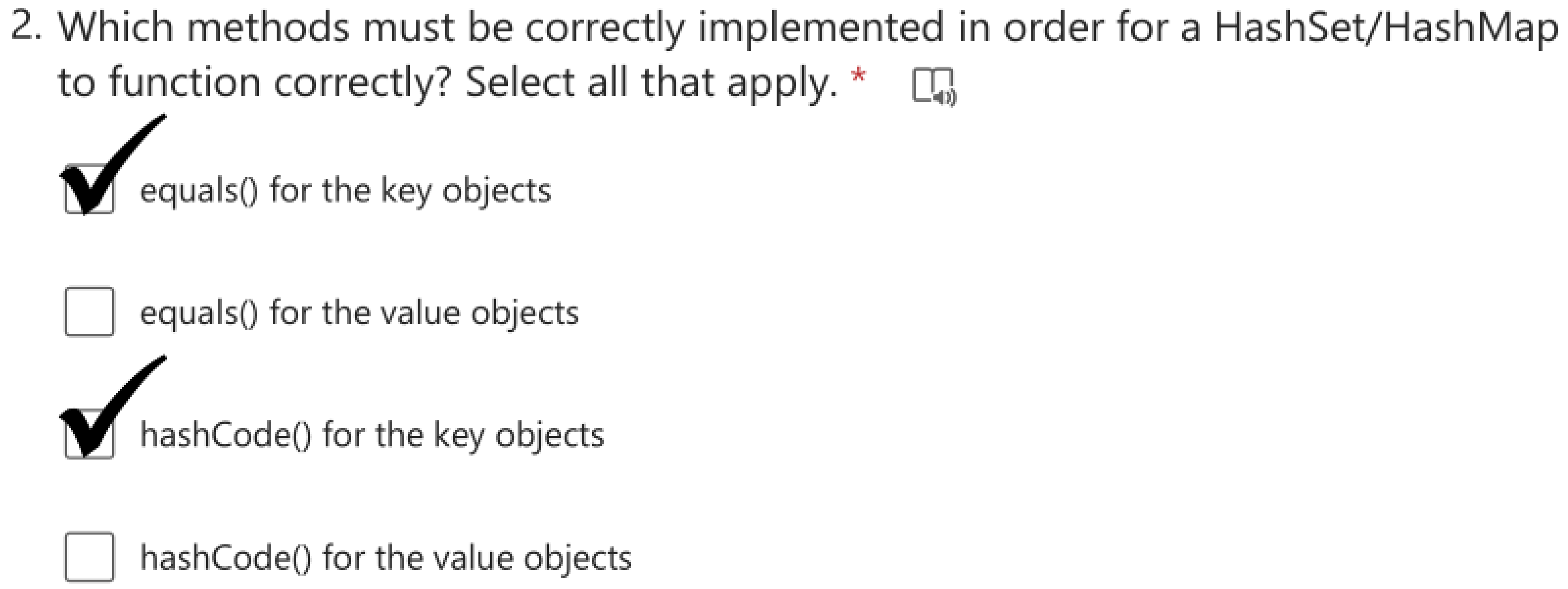
WOTO

Go to duke.is/v/syyy

Not graded for correctness, just participation.

Try to answer *without* looking back at slides and notes.

But do talk to your neighbors!



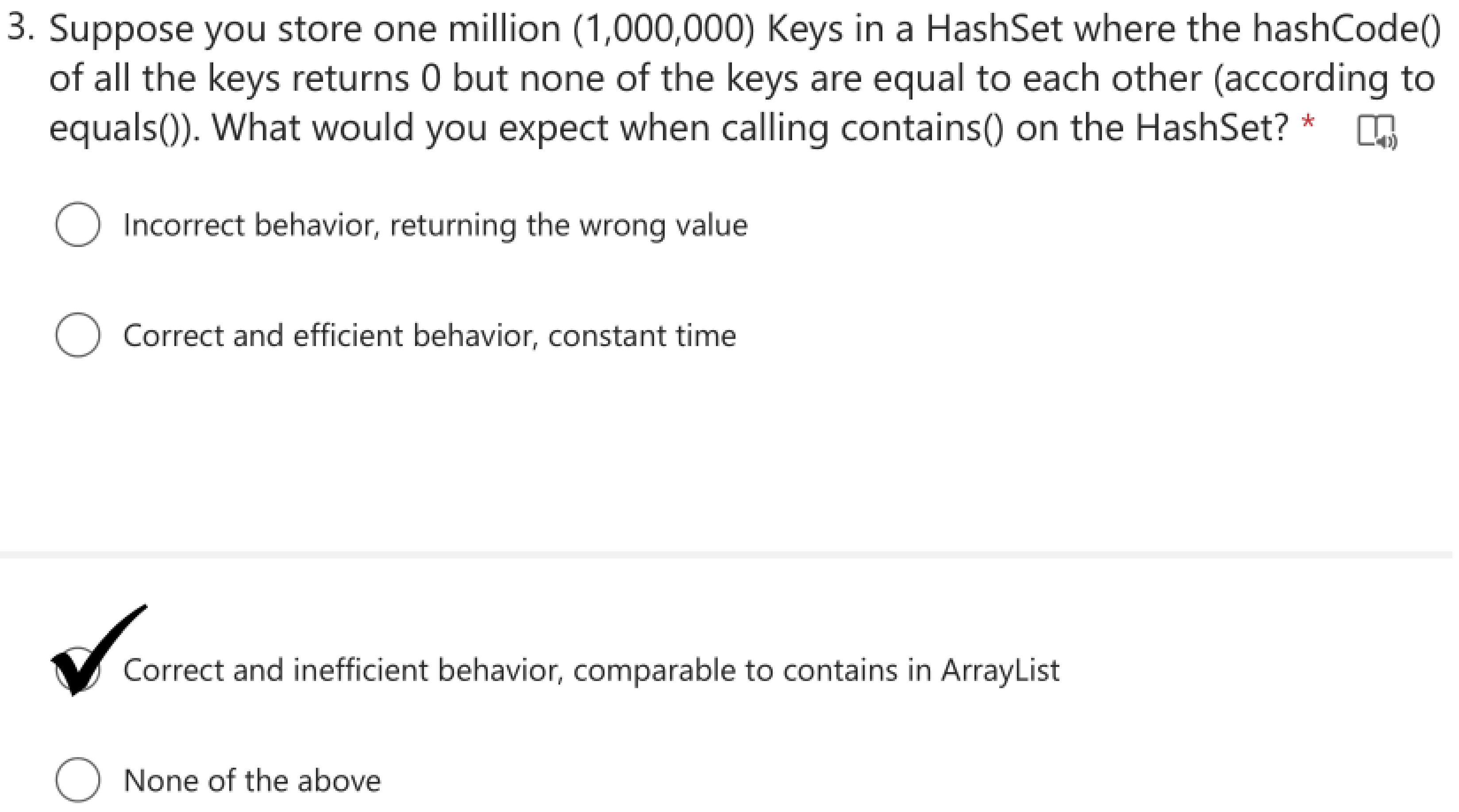
2. Which methods must be correctly implemented in order for a HashSet/HashMap to function correctly? Select all that apply. *

[✓ equals() for the key objects]

equals() for the value objects

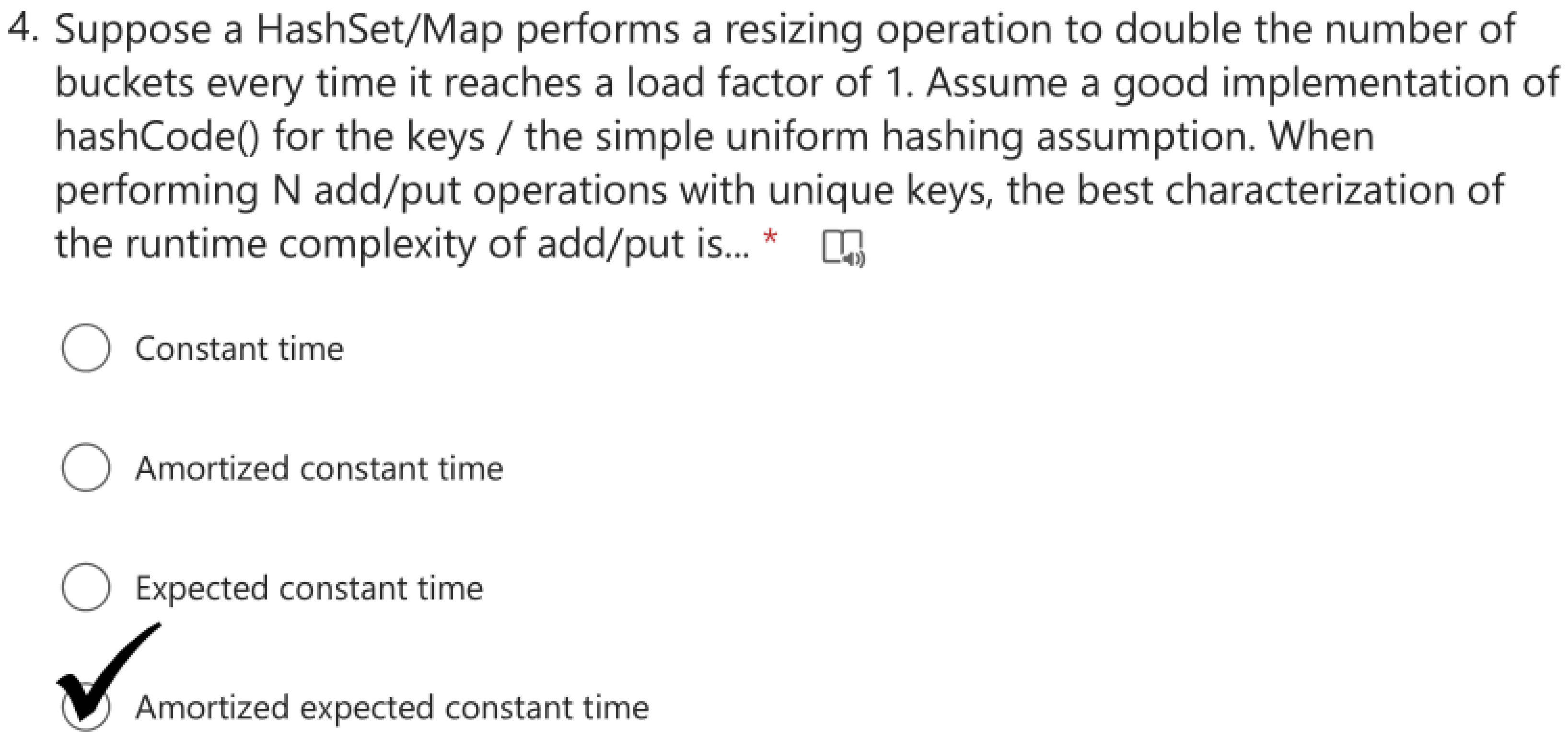
[✓ hashCode() for the key objects]

hashCode() for the value objects



3. Suppose you store one million (1,000,000) Keys in a HashSet where the hashCode() of all the keys returns 0 but none of the keys are equal to each other (according to equals()). What would you expect when calling contains() on the HashSet? *

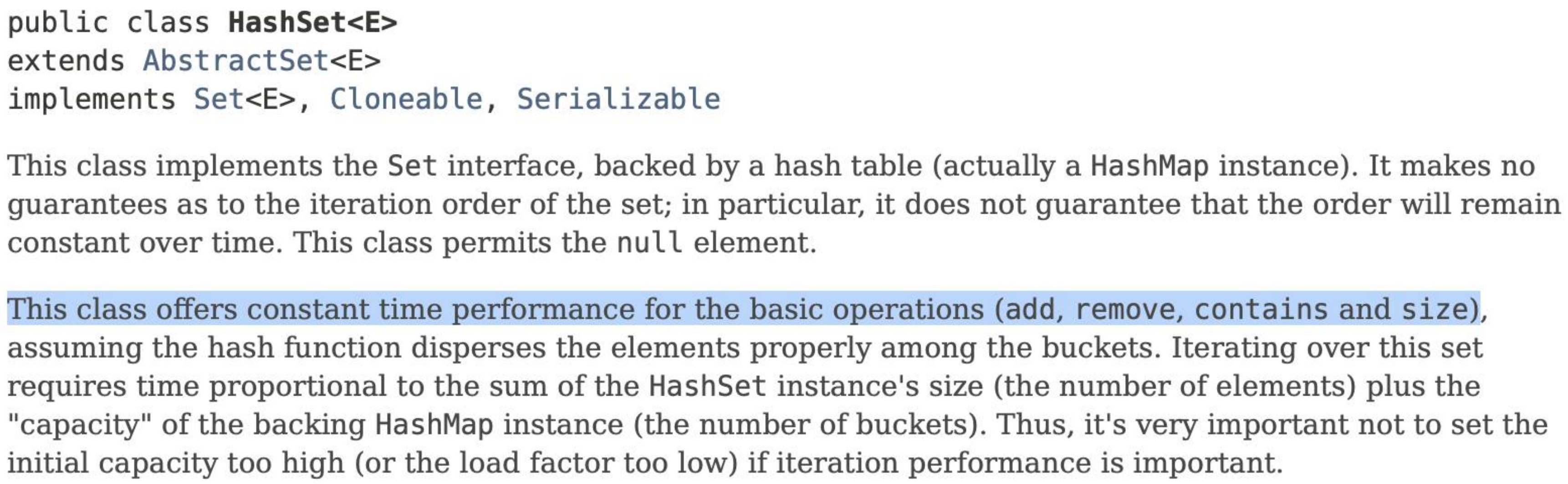
- ○ Incorrect behavior, returning the wrong value
- ○ Correct and efficient behavior, constant time
- [✓ Correct and inefficient behavior, comparable to contains in ArrayList]
- ○ None of the above



4. Suppose a HashSet/Map performs a resizing operation to double the number of buckets every time it reaches a load factor of 1. Assume a good implementation of hashCode() for the keys / the simple uniform hashing assumption. When performing N add/put operations with unique keys, the best characterization of the runtime complexity of add/put is... *

- Constant time
- Amortized constant time
- Expected constant time
- [✓ Amortized expected constant time]

Revisiting guarantees



```
public class HashSet<E>
extends AbstractSet<E>
implements Set<E>, Cloneable, Serializable
```

This class implements the `Set` interface, backed by a hash table (actually a `HashMap` instance). It makes no guarantees as to the iteration order of the set; in particular, it does not guarantee that the order will remain constant over time. This class permits the `null` element.

This class offers constant time performance for the basic operations (`add`, `remove`, `contains` and `size`), assuming the hash function disperses the elements properly among the buckets. Iterating over this set requires time proportional to the sum of the `HashSet` instance's size (the number of elements) plus the "capacity" of the backing `HashMap` instance (the number of buckets). Thus, it's very important not to set the initial capacity too high (or the load factor too low) if iteration performance is important.

Java API documentation

Constant ***amortized*** time operations ***in expectation*** under SUHA (practically: assuming the hash function distributes unequal keys).

Runtime Efficiency, an Empirical Look at String Concatenation

Two methods for repeated concatenation

```java
public static String repeatConcatA(int reps, String toConcat) {
    String result = new String();
    for (int i=0; i<reps; i++) {
        result += toConcat;
    }
    return result;
}
```

methodA: Using String object and basic **+** operator

```java
public static String repeatConcatB(int reps, String toConcat) {
    StringBuilder result = new StringBuilder();
    for (int i=0; i<reps; i++) {
        result.append(toConcat);
    }
    return result.toString();
}
```

methodB: Using `StringBuilder` object and **`append`** method

Empirical timing experiment

Can see the code on GitLab here.

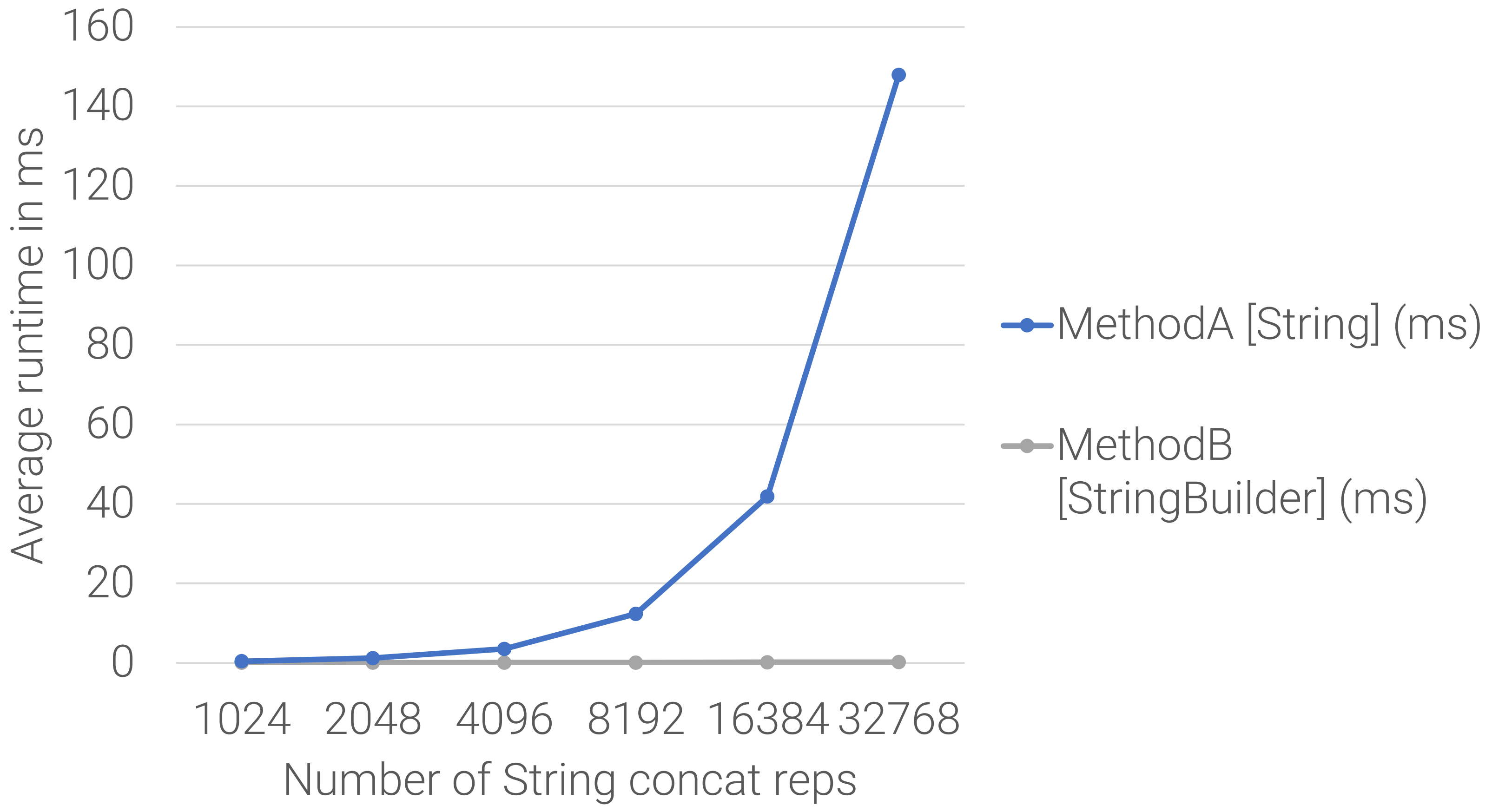
```
public class StringConcatTiming {
    static final int NUM_TRIALS = 100;
    static final int REPS_PER_TRIAL = 1024;
    static final String TO_CONCAT = "201";

    Run | Debug
    public static void main(String[] args) {
        long totalTime = 0;
        for (int trial=0; trial<NUM_TRIALS; trial++) {
            long startTime = System.nanoTime();
            //repeatConcatA(REPS_PER_TRIAL, TO_CONCAT);
            repeatConcatB(REPS_PER_TRIAL, TO_CONCAT);
            long endTime = System.nanoTime();
            totalTime += (endTime - startTime);
        }
        double avgTime = (double)totalTime / NUM_TRIALS;
        System.out.printf("Avg time per trial is %f ms", avgTime*1E-6);
    }
```

static final used for constants here

Going to time both methods separately.

Empirical results



Empirical results in more detail

Reps	MethodA (ms)	MethodB (ms)
1024	0.384	0.050
2048	1.136	0.061
4096	3.443	0.077
8192	12.244	0.099
16384	41.754	0.143
32768	147.719	0.207

Multiply reps by 2 multiplies runtime by 4. Quadratic complexity.

Multiply reps by 2 multiplies runtime by ~2. Linear complexity.

Empirical results in more detail

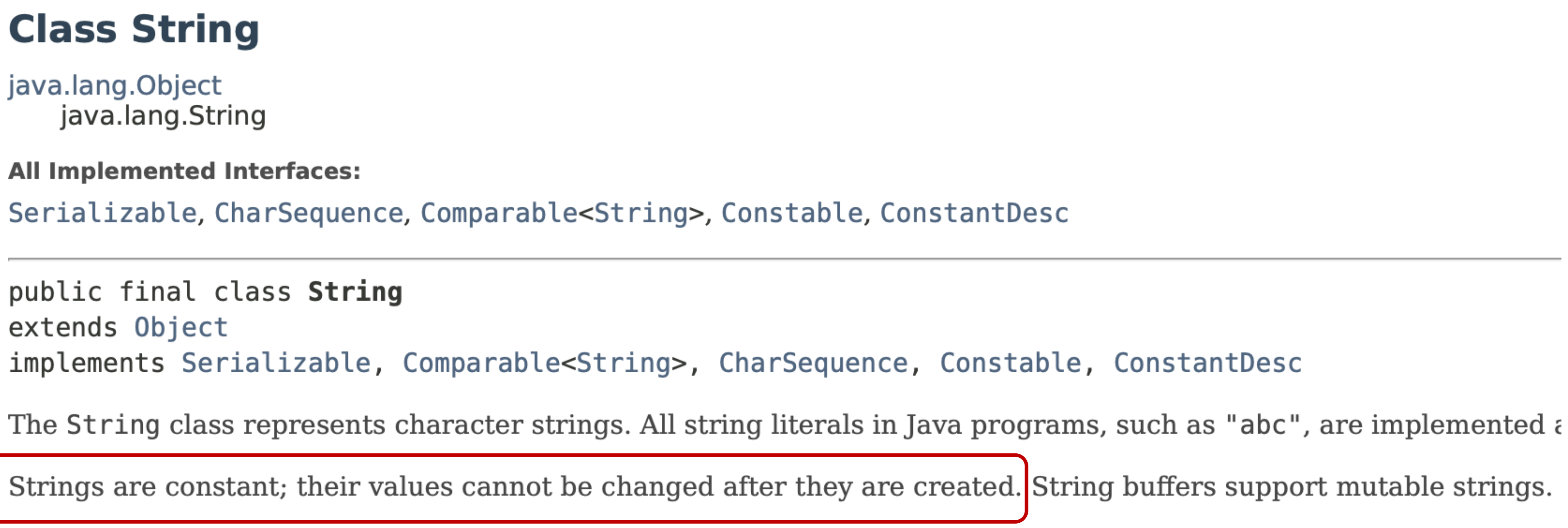
Reps	MethodA ns/rep	MethodB ns/rep
1024	0.375	0.048
2048	0.555	0.030
4096	0.841	0.019
8192	1.495	0.012
16384	2.548	0.009
32768	4.508	0.006

Runtime / rep increasing, *greater than linear* complexity.

Runtime / rep not increasing, at most linear complexity.

What’s going on? Documentation?

docs.oracle.com/en/java/javase/17/docs/api/java.base/java/lang/String



Class String

java.lang.Object
    java.lang.String

All Implemented Interfaces:

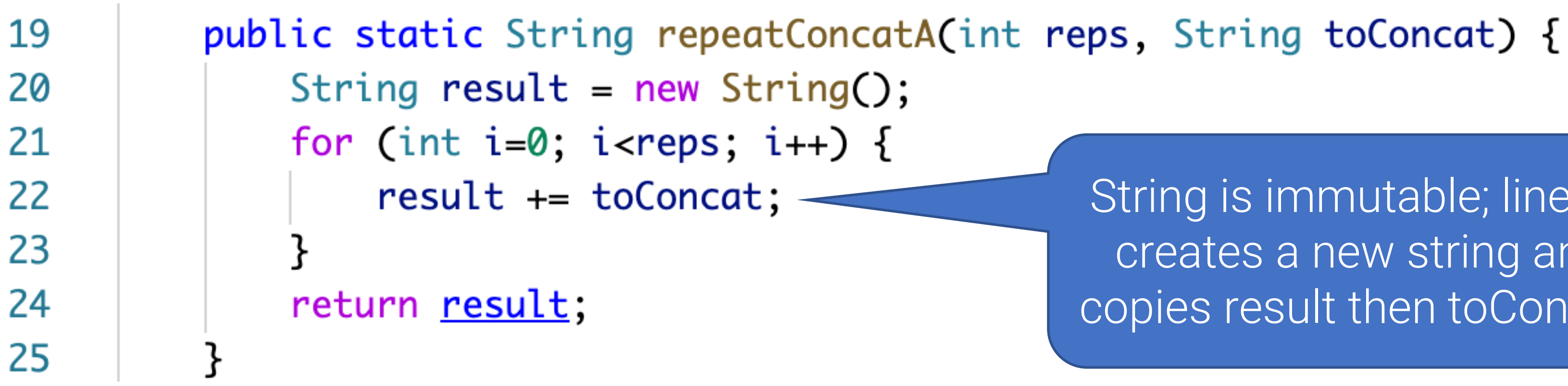
`Serializable, CharSequence, Comparable<String>, Constable, ConstantDesc`

```
public final class String
extends Object
implements Serializable, Comparable<String>, CharSequence, Constable, ConstantDesc
```

The `String` class represents character strings. All string literals in Java programs, such as `"abc"`, are implemented a

Strings are constant; their values cannot be changed after they are created. String buffers support mutable strings.

methodA revisited



```
19      public static String repeatConcatA(int reps, String toConcat) {
20          String result = new String();
21          for (int i=0; i<reps; i++) {
22              result += toConcat;
23          }
24          return result;
25      }
```

String is immutable; line 22 creates a new string and copies result then toConcat.

How many characters will be copied per iteration if `toConcat` is “`201`”?

- i=0: 3
- i=1: 6
- i=2: 9
- …
- On iteration i, need to copy 3*(i+1) characters!

How many total characters are copied? Algebra!

<u>`methodA`</u>: for `i` from `0` to `reps-1`,
copy `3*(i+1)` characters per iteration.

$$\sum_{i=0}^{\text{reps}-1} 3(\text{i}+1) = 3(\text{reps}) + 3\left(\sum_{i=0}^{\text{reps}-1} \text{i}\right)$$

$$= 3(\text{reps}) + 3\left(\frac{\text{reps}}{2}\right)(0 + \text{reps} - 1)$$

$$= \frac{3}{2}(\text{reps}^2 + \text{reps})$$

Arithmetic series formula:

$$\sum_{i=1}^{n} a_i = \left(\frac{n}{2}\right)(a_1 + a_n)$$

Abstracting, Intro to Big O Notation (Preview for next time)

- The 3/2 in $\frac{3}{2}\mathbf{reps}^2$ doesn't tell us much about how the performance *scales with the size of reps*.

- Often, we use ***asymptotic notation***, especially ***Big O notation*** to abstract away constants.

- For example: Let N = reps, then we say that the asymptotic runtime complexity is O(N^2).
  - If you ~double N, you ~quadruple the runtime

Two general Big O rules

1. Can drop constants
   - $2N+3 \Rightarrow O(N)$
   - $0.001N + 1{,}000{,}000 \Rightarrow O(N)$

2. Can drop lower order terms
   - $2N^2+3N \Rightarrow O(2N^2) \Rightarrow O(N^2)$
   - $N+\log(N) \Rightarrow O(N)$
   - $2^N + N^2 \Rightarrow O(2^N)$

Hierarchy of some common complexity classes

Big O	Name	Example
$O(2^N)$	Exponential	Calculate all subsets of a set
$O(N^3)$	Cubic	Multiply NxN matrices
$O(N^2)$	Quadratic	Loop over all *pairs* from N things
O(N log(N))	Nearly-linear	Sorting algorithms
O(N)	Linear	Loop over N things
O(log(N))	Logarithmic	Binary search a sorted list
O(1)	Constant	Addition, array access, etc.

How does StringBuilder work?

"Every string builder has a capacity. As long as the length of the character sequence contained in the string builder does not exceed the capacity, it is not necessary to allocate a new internal buffer. If the internal buffer overflows, it is automatically made larger." - StringBuilder JDK 17 documentation.

- But how does it grow?
- Geometrically! Like ArrayList, HashMap, …
  - Still linear amortized complexity, for same reasons

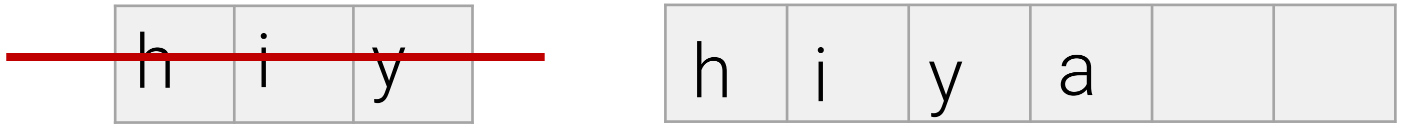
StringBuilder is like an ArrayList of characters

- Suppose we run the code:

```
StringBuilder() sb = new StringBuilder(3);
sb.append(“hi”);
sb.append(“ya”);
```

Initial buffer/array capacity

Array representing StringBuilder



How many total characters are copied with a StringBuilder?

Suppose we

- start with capacity 3,
- append a length 3 string **reps** times, and
- double when out of capacity.

$$3 \cdot \mathbf{reps} + \sum_{i=0}^{\approx \log_2(3 \cdot \mathbf{reps})} 2^i = 3 \cdot \mathbf{reps} + 6 \cdot \mathbf{reps}$$

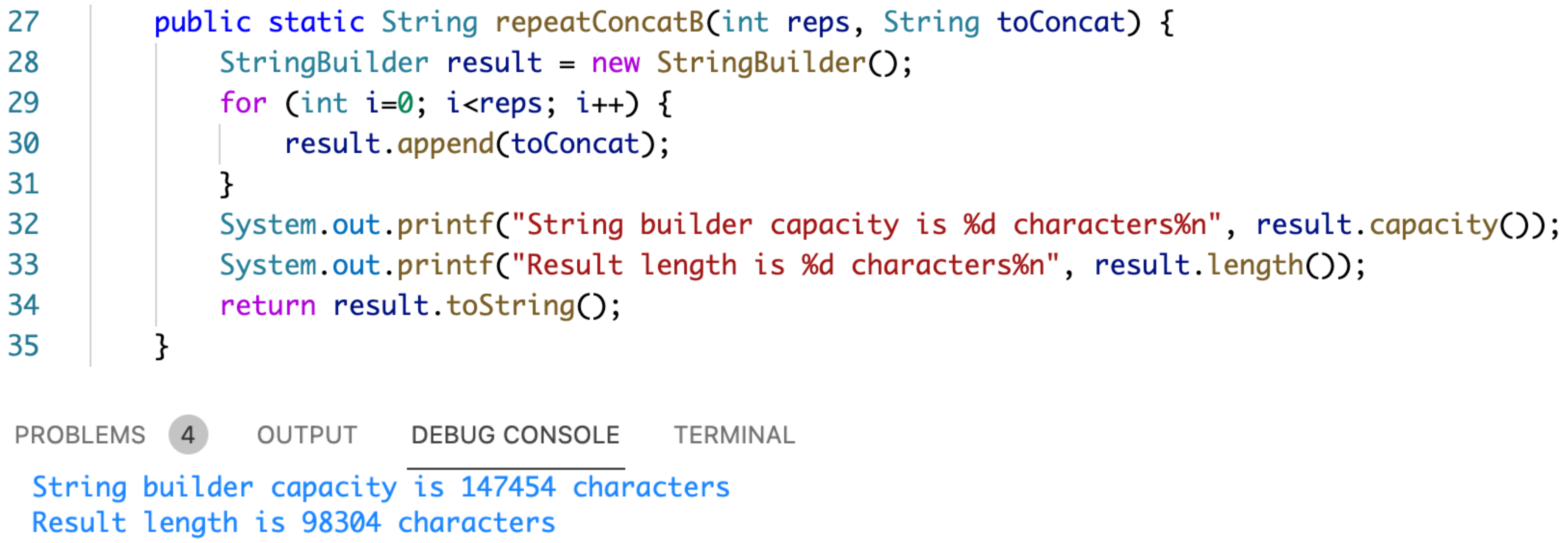
The "good case" copies

From doubling and copying the array

Geometric series formula: $\sum_{i=0}^{n} a\, r^i = a\left(\frac{1 - r^{n+1}}{1 - r}\right)$

$$= 9 \cdot \mathbf{reps}$$

Memory/Runtime Tradeoff



```java
public static String repeatConcatB(int reps, String toConcat) {
    StringBuilder result = new StringBuilder();
    for (int i=0; i<reps; i++) {
        result.append(toConcat);
    }
    System.out.printf("String builder capacity is %d characters%n", result.capacity());
    System.out.printf("Result length is %d characters%n", result.length());
    return result.toString();
}
```

```
String builder capacity is 147454 characters
Result length is 98304 characters
```

Final StringBuilder is using about 146k / 98k ~= 1.5 times as much memory as necessary. Very common tradeoff in data structures!

What's the real difference between methodA and methodB?

- methodA: Copies roughly $\frac{3}{2}(\mathbf{reps}^2 - \mathbf{reps})$
- methodB: copies roughly $9 \cdot \mathbf{reps}$ characters.

Reps	~MethodA char copies (millions)	MethodB char copies (millions)
1000	1.5	0.009
2000	6	0.018
4000	24	0.036
8000	95	0.072
16000	383	0.144
32000	1535	0.288

WOTO

Go to duke.is/m/pm9u

Not graded for correctness, just participation.

Try to answer *without* looking back at slides and notes.

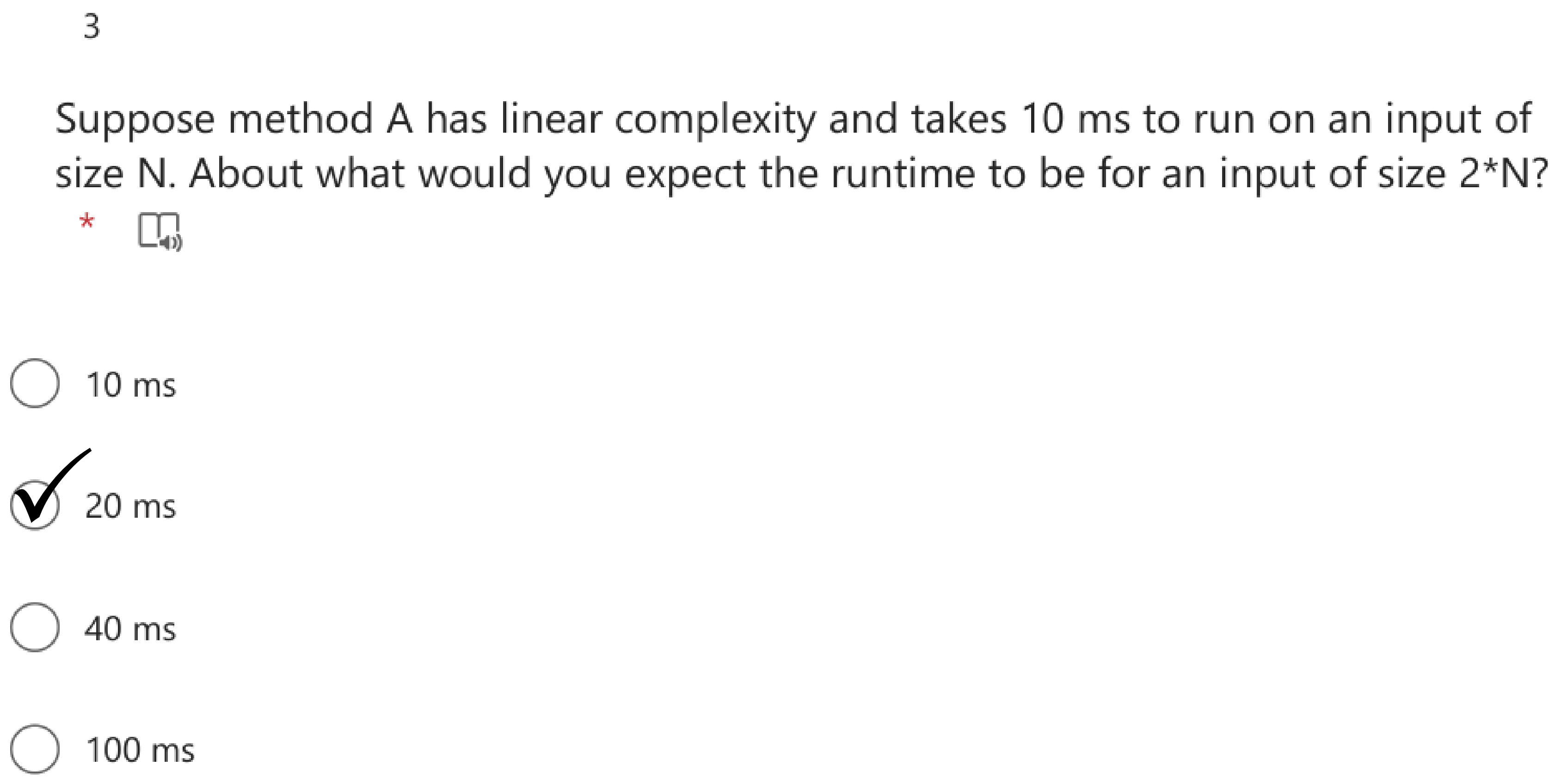
But do talk to your neighbors!

2

How many **total characters must be copied** by the code **on lines 8 and 9**? Remember that Strings are immutable in Java. *

```
7        String s = "hi";
8        s += "hey";
9        s += s;
```

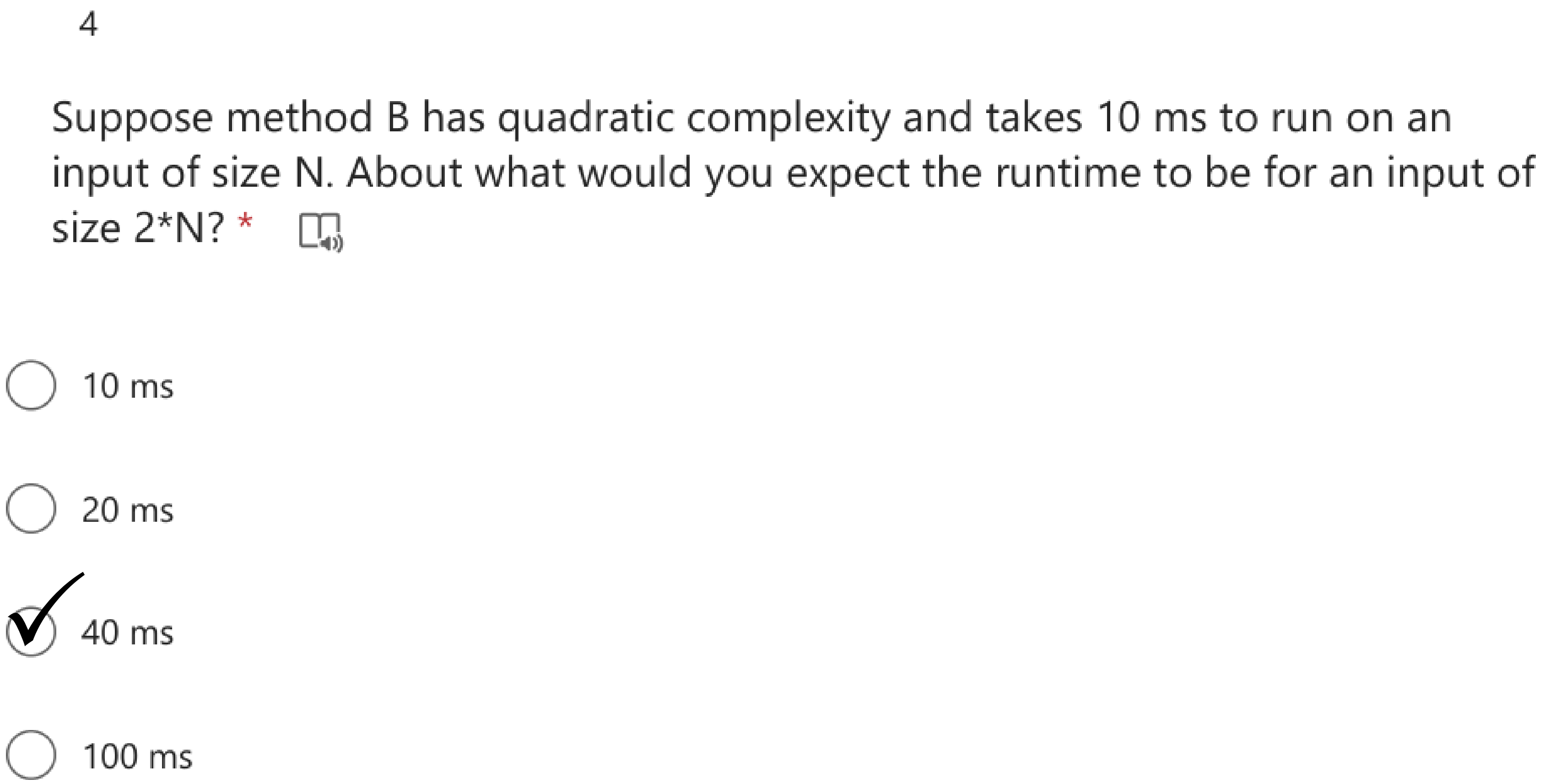
- 5
- 7
- 9
- 10
- [✓ 15]
- 30



3

Suppose method A has linear complexity and takes 10 ms to run on an input of size N. About what would you expect the runtime to be for an input of size 2*N?
*

- 10 ms
- [✓ 20 ms]
- 40 ms
- 100 ms



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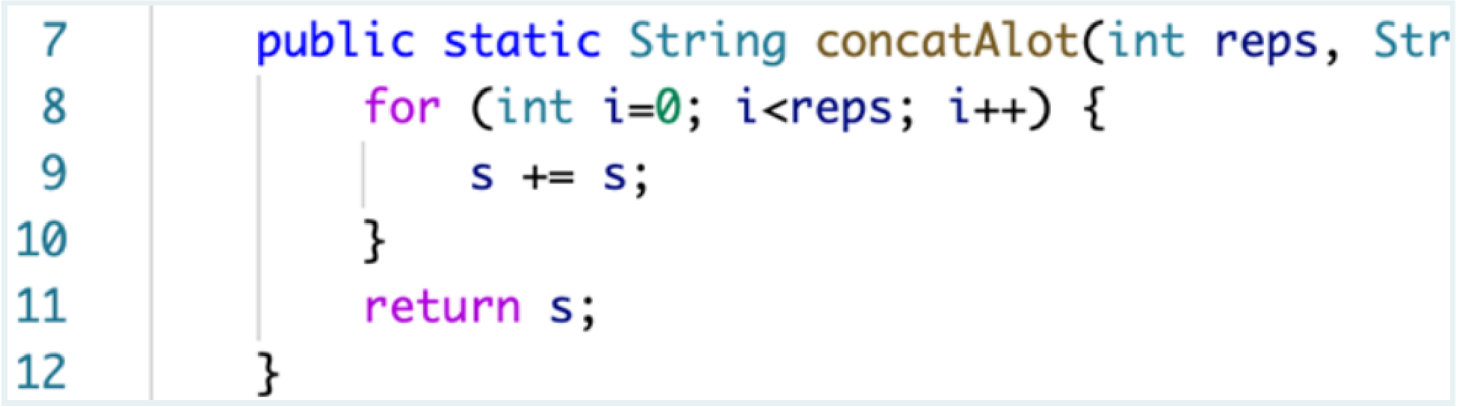
Suppose method B has quadratic complexity and takes 10 ms to run on an input of size N. About what would you expect the runtime to be for an input of size 2*N? *

- 10 ms
- 20 ms
- [✓ 40 ms]
- 100 ms

Here is another String concatenation method. Suppose the input string s has a small number of characters, say 3. As a function of the parameter reps, how

would you characterize the runtime complexity of the method? Hint: As a function of reps, how many total characters will be copied across all iterations of the loop? *



```
    public static String concatAlot(int reps, Str
        for (int i=0; i<reps; i++) {
            s += s;
        }
        return s;
    }
```

- Constant
- Linear
- Quadratic
- [✓ Exponential]