L13: Recursion

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CompSci 201: Spring 2024

2/26/2024

Person in CS: Ellen Ochoa

- BS physics (‘75), PhD Elec. Eng. (‘85)
- Starting working on software for optical recognition systems (computer vision)
- Applied to be an astronaut in…
  - ‘85…rejected
  - ‘87…rejected
  - ‘90…accepted!!!
- Worked on flight software, computer hardware, and robotics
- First Hispanic woman in space ’93
- Director of NASA Johnson Space Flight Center (Houston) ‘13

Announcements, Coming up

- Today, Monday 2/26
  - Nothing due
- Wednesday 2/28
  - APT 5 (linked list problems) due
- Next Monday 3/4
  - Project P3: DNA (linked list project) due

Today's outline

- Introducing Recursion
  - Counting ListNodes
  - Reversing a LinkedList
- Live Coding w/ Recursion
  - (Time permitting)

Toward Recursion by counting nodes: Iterative vs. Recursive

- Standard linked list iteration
  - Advance local pointer, do something at each node

```
public int countIter(ListNode list) {
    int total = 0;
    while (list != null) {
        total += 1;
        list = list.next;
    }
    return total;
}
```

- Recursion?
  - Base Case?
  - General case?
  - Define `size` in terms of `size`?

```
public int size(ListNode list) {
    if (list == null) return 0;
    return 1 + size(list.next);
}
```

Key ideas in recursion

1. Base case: Solve for answer when instance is "small"
2. General case:
    1. Get answer on ***smaller*** instance(s) of the ***same*** problem using recursive call(s)
    2. Do something with the result of the recursive call(s) and then return

- Note: Methods/calls stacked, like all methods

Thinking recursively

1. When is the input small enough that the answer is trivial? Base case.

   Base case

2. Otherwise, suppose a magical fairy (*the Recursion Fairy!*) could solve the ***exact same problem*** on a ***smaller*** input

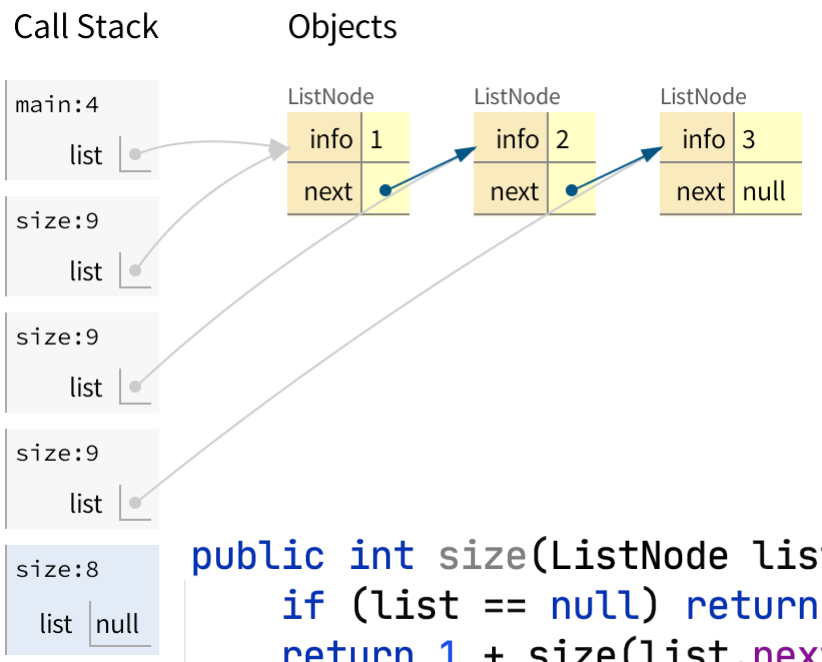
   Recurse

3. Could you solve the larger problem given what the fairy told you?

   Use result

The call stack: How recursion works on a machine

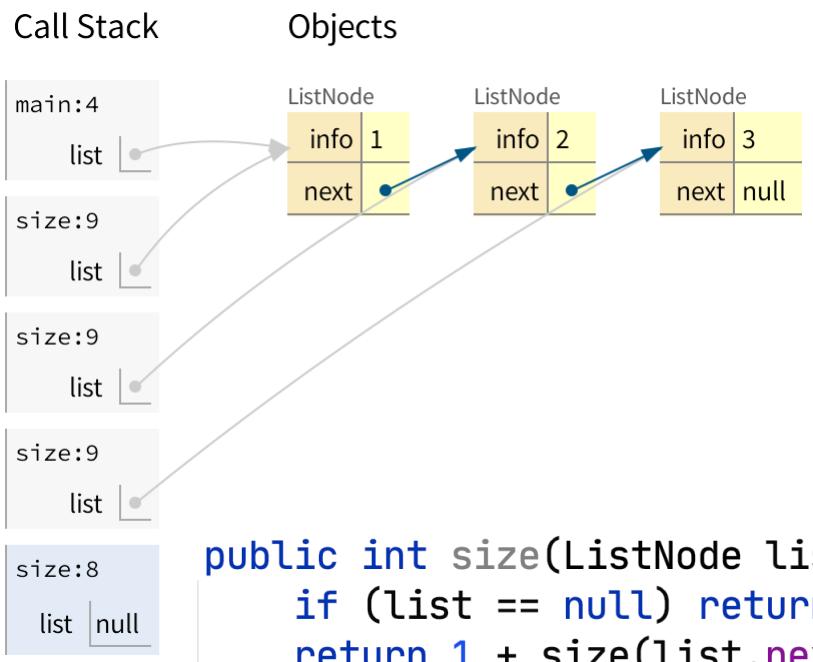
- Each method call gets its own ***call frame*** (local variables, etc.)
- ***Eager evaluation***: Invoking method does not resume until invoked method returns.




```
public int size(ListNode list) {
    if (list == null) return 0;
    return 1 + size(list.next);
}
```

Eager evaluation and substitution

- Return value will be substituted into the expression calling the method.

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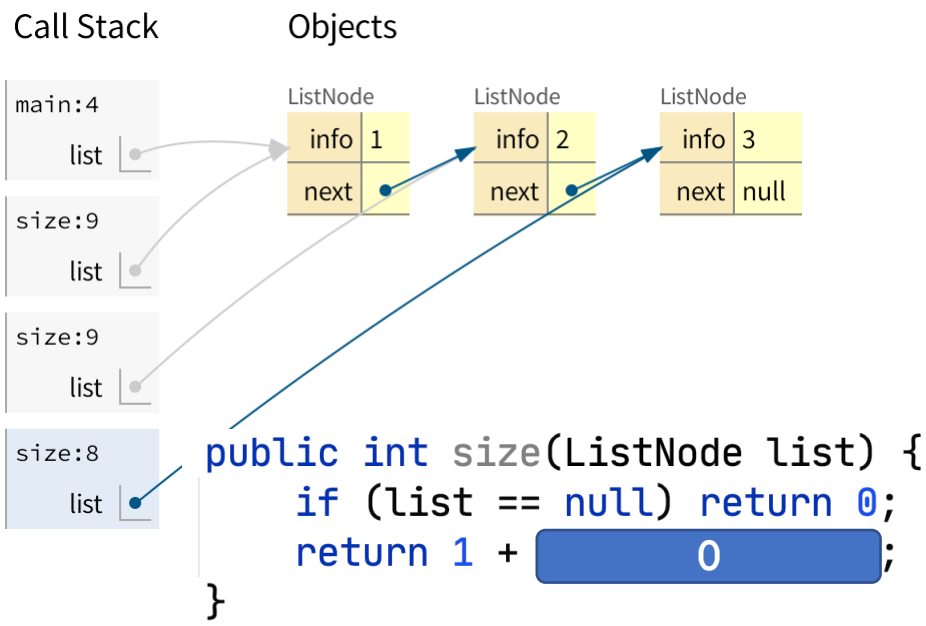

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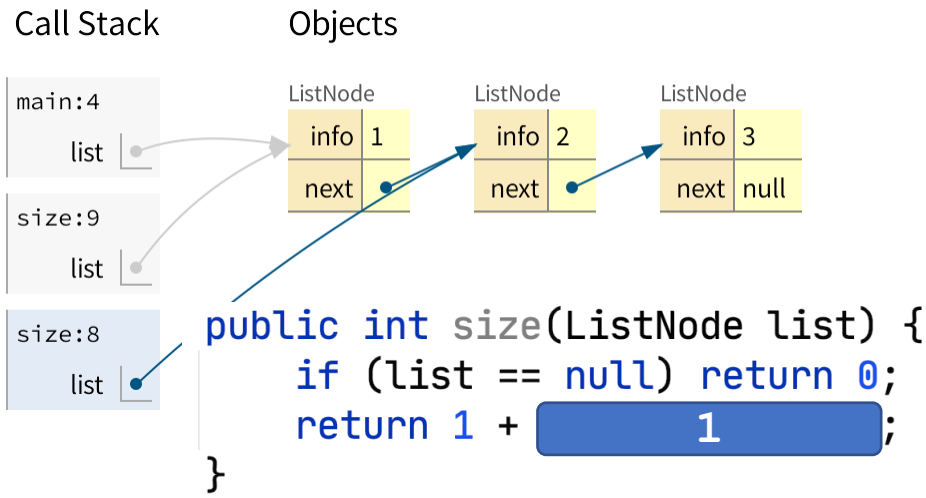

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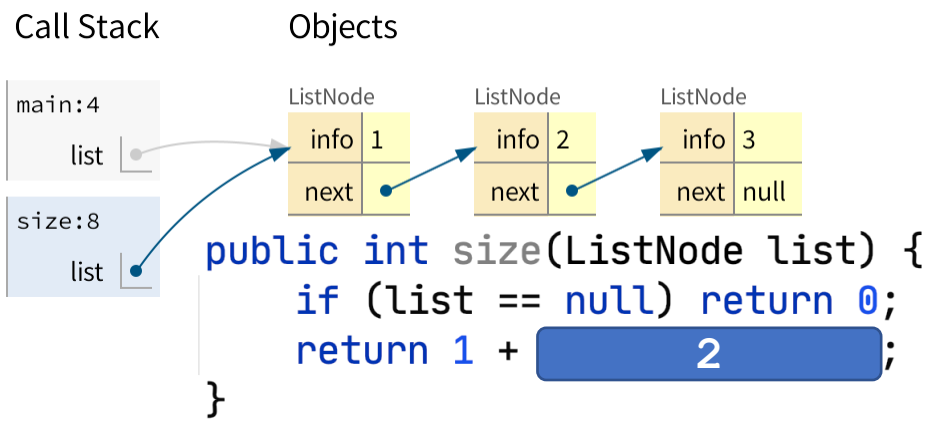
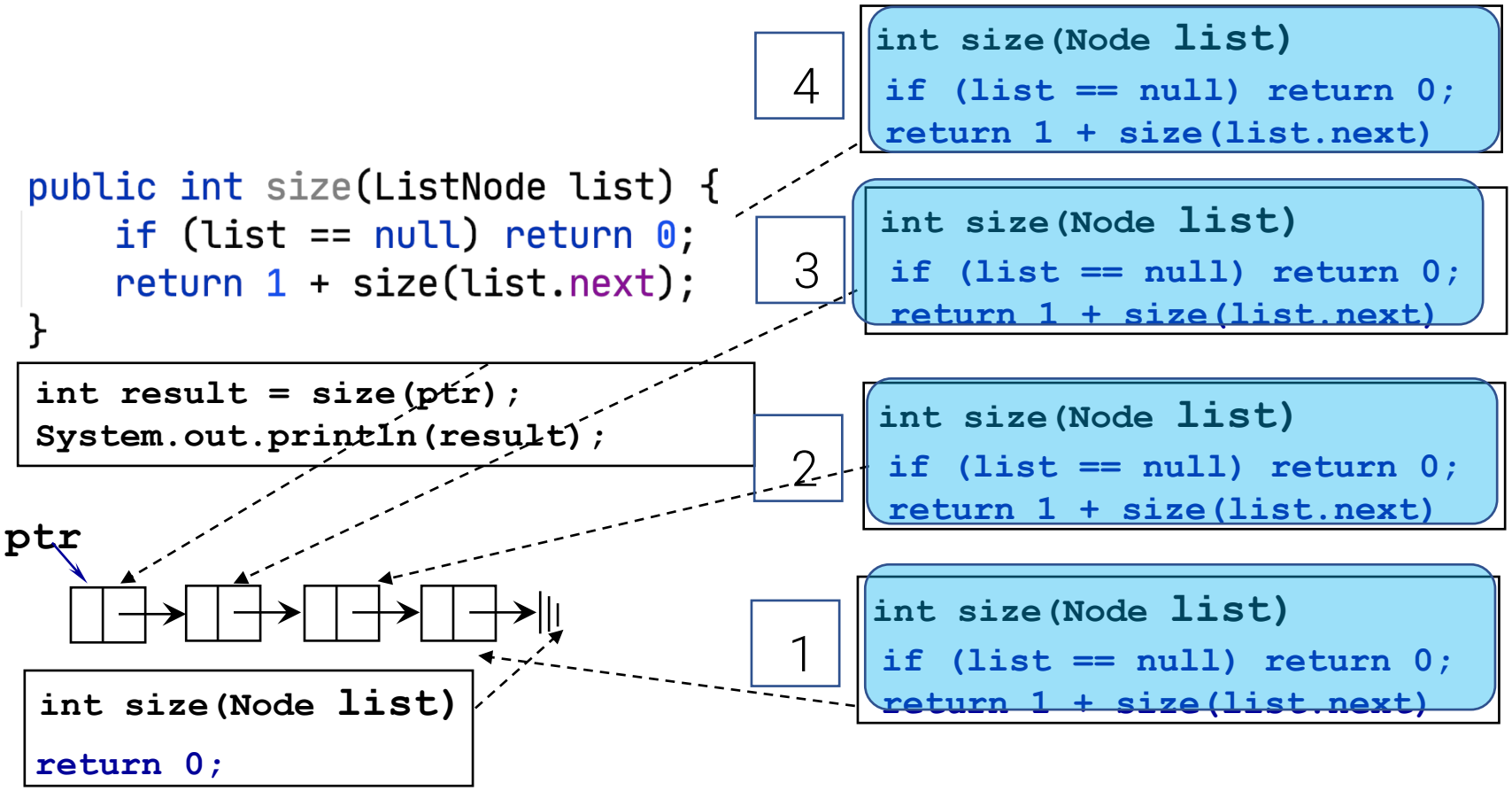


Counting Nodes



Recursive runtime

- Concept is the same: Count the number of constant time operations…across all recursive calls!

- *Ensure each recursive call gets closer to the base case*, else code may run forever.

```
public int size(ListNode list) {
    if (list == null) return 0;
    return 1 + size(list.next);
}
```

- Moves one node toward the base case at each step.
- List of N nodes, makes O(N) total recursive calls, each takes O(1) time
- Overall O(N) runtime complexity.

Recall the reverse problem

- How do we reverse nodes in a linked list
  - Go from A->B->C to C->B->A
  - Typical interview style question
  - https://leetcode.com/problems/reverse-linked-list/
  - https://www.hackerrank.com/challenges/reverse-a-linked-list

Base case, words and code

- Base case: When is there nothing to do?
  - A list with 0 or 1 nodes is its own reverse

```
3    public static ListNode reverse(ListNode list) {
4        if (list == null || list.next == null) {
5            return list;
6        }
```

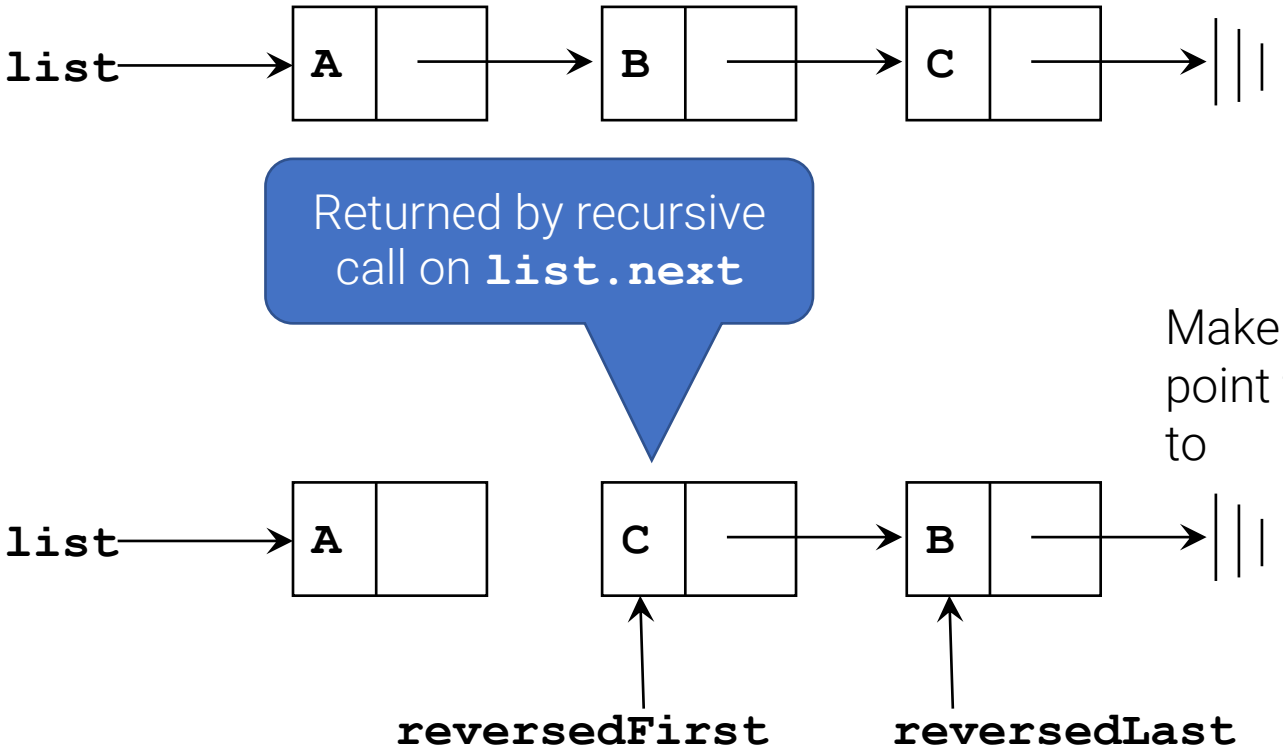
Recursive step in words

- Suppose the Recursion Fairy (a recursive call) reverses the list *after the first node*.

- How to use? Just put the first node at the end!

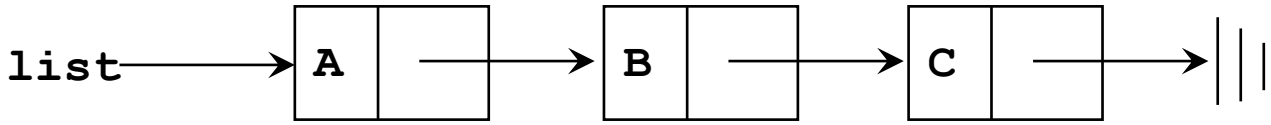
- Restated: The reverse of a list is ***the reverse of all but the first element***, with the first element added to the end.

Recursive step in pictures

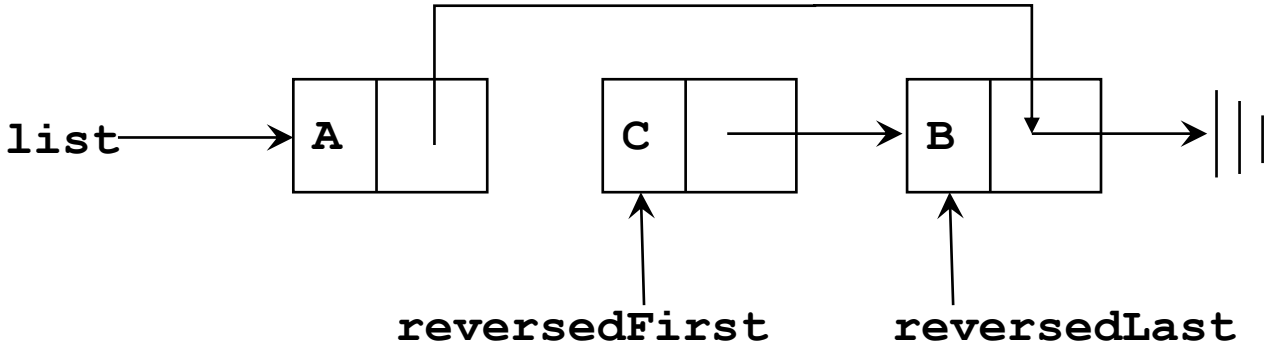


Return **reversedFirst**

Recursive step in code

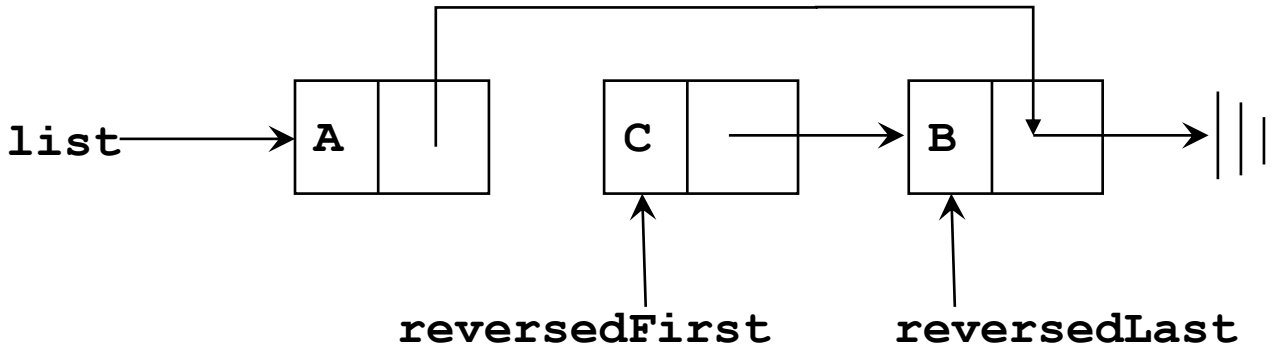



```
7       ListNode reversedLast = list.next;
8       ListNode reversedFirst = reverse(list.next);
```



Note that **`list.next`** still refers to **`reversedLast`**

Recursive step in code (continued)

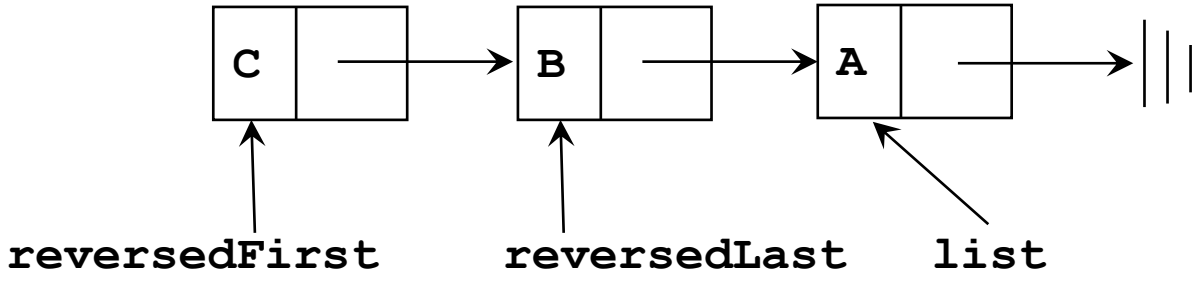



```
 9        reversedLast.next = list;
10        list.next = null;
11        return reversedFirst;
```

Make **B** point to **A**

Make **A** point to **null**

Return overall reversed list



Putting it all together

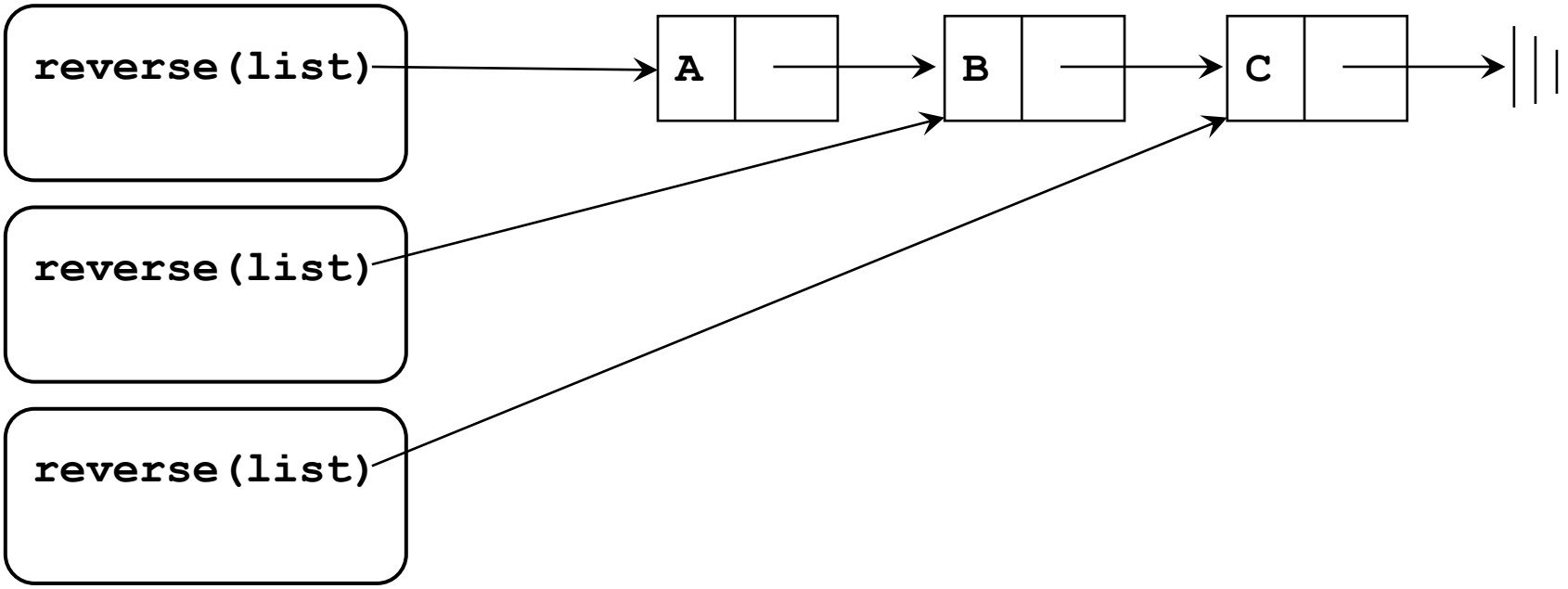
```
3   public static ListNode reverse(ListNode list) {
4       if (list == null || list.next == null) {
5           return list;
6       }
7       ListNode reversedLast = list.next;
8       ListNode reversedFirst = reverse(list.next);
9       reversedLast.next = list;
10      list.next = null;
11      return reversedFirst;
12  }
```

Base case

Recurse

Use result

Revisiting the call stack: How it *really* works



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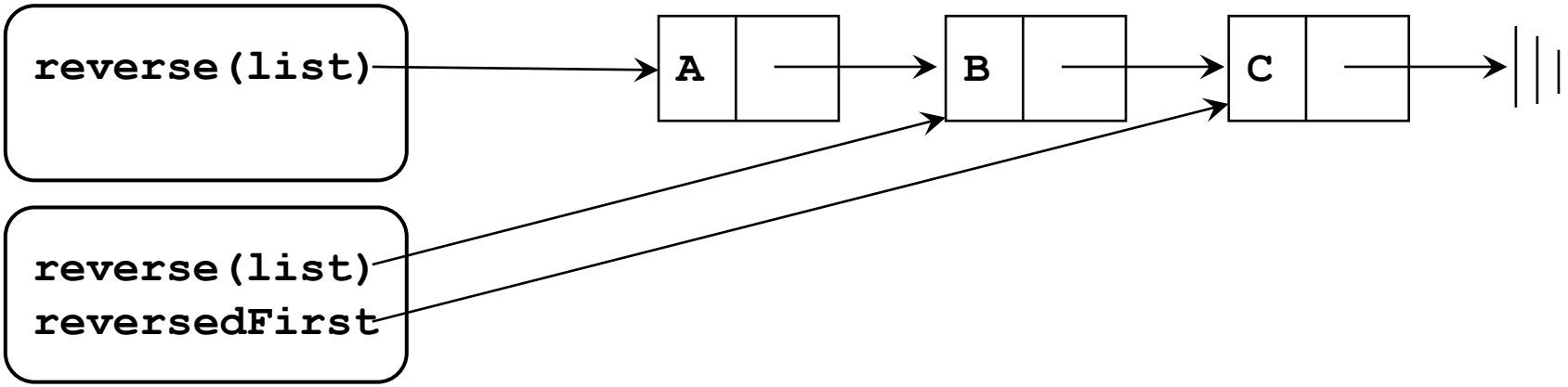

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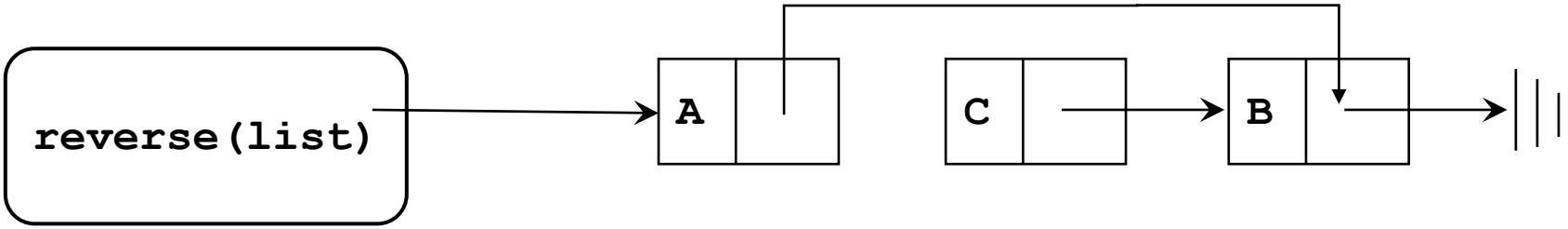

Back to the case we considered first

Consider the rec method. If the input list is ['A', 'B', 'C'], what will be returned by rec(list)?

```
3   public static ListNode rec(ListNode list) {
4       if (list == null || list.next == null) {
5           return list;
6       }
7       ListNode after = rec(list.next);
8       if (list.info <= after.info) {
9           list.next = after;
10          return list;
11      }
12      return after;
13  }
```

Answer: [‘A’, ‘B’, ‘C’]

If the input list is ['C', 'B', 'A'], what will be returned by rec(list)?

```
3   public static ListNode rec(ListNode list) {
4       if (list == null || list.next == null) {
5           return list;
6       }
7       ListNode after = rec(list.next);
8       if (list.info <= after.info) {
9           list.next = after;
10          return list;
11      }
12      return after;
13  }
```

Answer: [‘A’]

For an input list with N nodes, the best characterization of the runtime complexity of rec(list) is...

```
3   public static ListNode rec(ListNode list) {
4       if (list == null || list.next == null) {
5           return list;
6       }
7       ListNode after = rec(list.next);
8       if (list.info <= after.info) {
9           list.next = after;
10          return list;
11      }
12      return after;
13  }
```

Answer: O(N)

Consider the mystery method. Note that it is the same as rec except for lines 24-29. If the input list is ['C', 'B', 'A'], what will be returned by mystery(list)?

```
15  public static ListNode mystery(ListNode list) {
16      if (list == null || list.next == null) {
17          return list;
18      }
19      ListNode after = mystery(list.next);
20      if (list.info <= after.info) {
21          list.next = after;
22          return list;
23      }
24      ListNode current = after;
25      while (current.next != null && list.info > current.next.info) {
26          current = current.next;
27      }
28      list.next = current.next;
29      current.next = list;
30      return after;
31  }
```

Answer: [‘A’, ‘B’, ‘C’]

Same mystery method. For an input list with N nodes, the best characterization of the runtime complexity of mystery(list) is...

```
15  public static ListNode mystery(ListNode list) {
16      if (list == null || list.next == null) {
17          return list;
18      }
19      ListNode after = mystery(list.next);
20      if (list.info <= after.info) {
21          list.next = after;
22          return list;
23      }
24      ListNode current = after;
25      while (current.next != null && list.info > current.next.info) {
26          current = current.next;
27      }
28      list.next = current.next;
29      current.next = list;
30      return after;
31  }
```

Answer: O(N^2)