L20: Binary Heaps

Alex Steiger

CompSci 201: Spring 2024

3/27/2024

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People in CS: Clarence “Skip” Ellis

- Born 1943 in Chicago. PhD in CS from UIUC in 1969
  - First African American in US to complete a PhD in CS
- Founding member of the CS department at U. Colorado, also worked in industry.
  - Developing original graphical user interfaces, object-oriented programming, collaboration tools.

“People put together an image of what I was supposed to be,” he recalled. “So I always tell my students to push.”

Read more here

2

Logistics, Coming up

- Today, Wednesday 3/29
  - APT 7 due
- Next Monday, 4/3
  - Nothing due, start on P5 Huffman
- Next Wednesday, 4/5
  - APT 8 due

3

Today's agenda

- Wrap up Huffman Coding Intro
- Priority Queue revisited
  - Implementations, especially binary heap

4

Huffman Compression

Representing data with bits: Preferably fewer bits

- Zip 
- Unicode 

- JPEG 
- MP3 

Huffman compression used in all of these and more!

5

Decoding Variable Length

- What if we use
  - a = 1
  - b = 10
  - c = 11
- How would we decode 1011?
  - "baa" or "bc?"
- Problem: Encoding of a (1) is a ***prefix*** of the encoding for c (11). Ambiguous!

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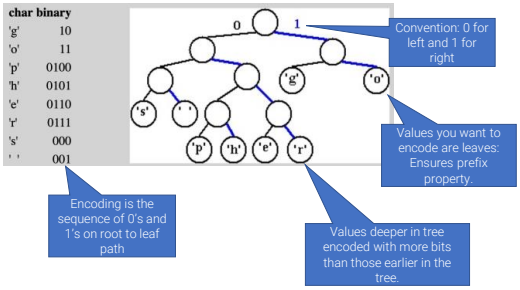
Prefix Property: Encoding as a Tree



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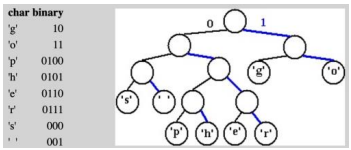
Decoding bits using Huffman tree

Goal: Decode 10011011 assuming it was encoded with this tree.



- Read bit at a time, traverse left or right edge.
- When you reach a leaf, decode the character, restart at root.

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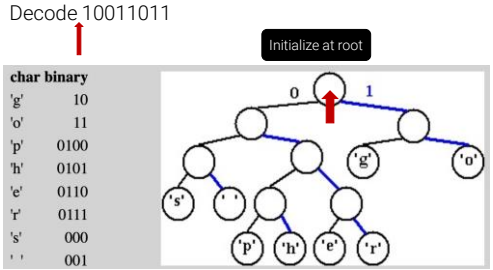
Decoding bits using Huffman tree

Decode 10011011



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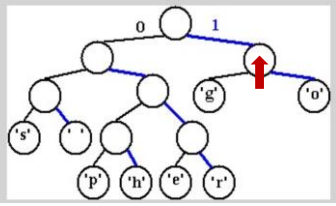
Decoding bits using Huffman tree

Decode 10011011

Read 1, go to right child

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



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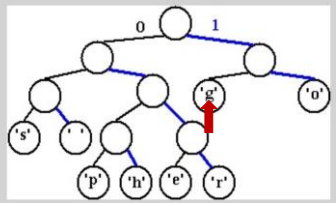
Decoding bits using Huffman tree

Decode 10011011

Read 0, go to left child

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



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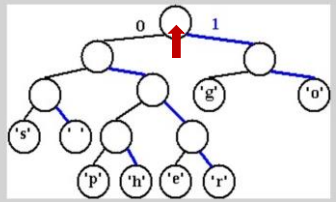
Decoding bits using Huffman tree

Decode 10011011

g

Leaf, decode 'g', restart at root

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



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Decoding bits using Huffman tree

Decode 10011011

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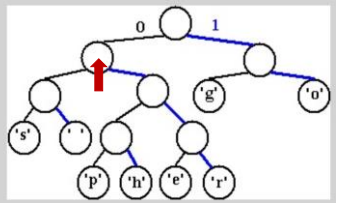

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Decoding bits using Huffman tree

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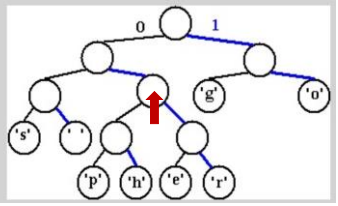

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Decoding bits using Huffman tree

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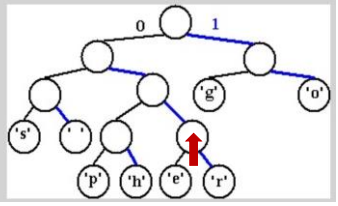

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Decoding bits using Huffman tree

Decode 10011011

g

Read 0, go to left child

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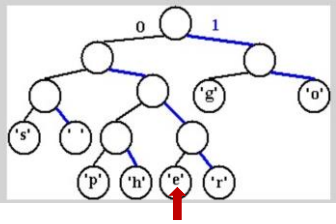
's'	000
' '	001



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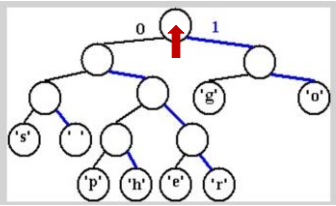
Decoding bits using Huffman tree

Decode 10011011

ge

Leaf, decode 'e', restart at root

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



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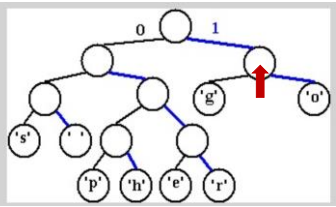
Decoding bits using Huffman tree

Decode 10011011

ge

Read 1, go to right child

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



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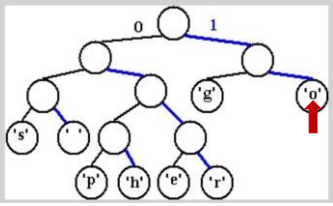
Decoding bits using Huffman tree

Decode 10011011

ge

Read 1, go to right child

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



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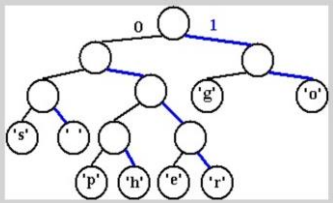
Decoding bits using Huffman tree

Decode 1001 1011

geo

Leaf, decode 'o'

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



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Huffman Coding

- ***Greedy*** algorithm for building an optimal variable-length encoding tree.
- High level idea:
  - Start with the leaves/values you want to encode with weights = frequency. Then repeat until all leaves are in single tree:
  - ***Greedy*** step: Choose the ***lowest-weight nodes*** to connect as children to a new node with weight = sum of children.
- Implementation? Use a priority queue!

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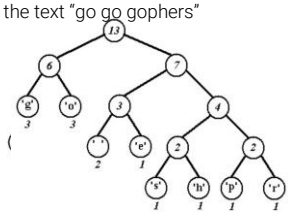
Visualizing the greedy algorithm

Encoding the text "go go gophers"



char	binary
'g'	00
'o'	01
'p'	1110
'h'	1101
'e'	101
'r'	1111
's'	1100
' '	100

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P5 Outline

1. Write Decompress first
   - Takes a compressed file (we give you some)
   - Reads Huffman tree from bits
   - Uses tree to decode bits to text
2. Write Compress second
   - Count frequencies of values/characters
   - Greedy algorithm to build Huffman tree
   - Save tree and file encoded as bits

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Priority Queues Revisited, Binary Heaps

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L20-WOTO1-Huffman-Sp24

Hi, Alexander. When you submit this form, the owner will see your name and email address.

* Required

1

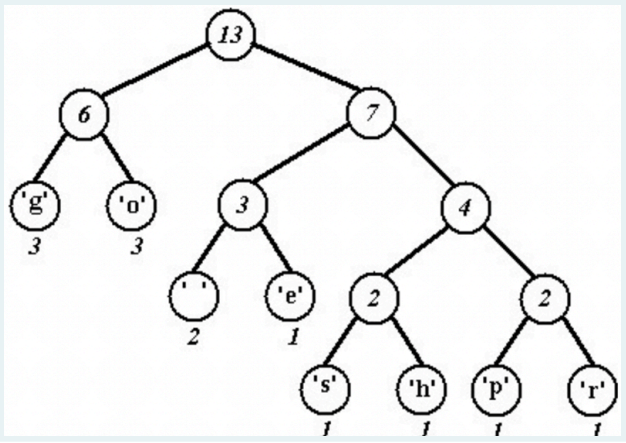
NetID *

solutions

2

Given the Huffman coding tree shown, what is the decoded text corresponding to the compressed bit sequence "**1101 0111 1111 0010 1**"?

These bits have been shown in blocks of 4 for readability; that does **not** mean each 4 bits codes for a single character. *



horse

3

Given these frequencies, how long will the encoding for 'a' be? How long will the encoding for 'b' be? *

Character	Frequency
a	30
b	20
c	10
d	15
e	40

- ○ 'a' -> 1 bit, 'b' -> 1 bit
- ○ 'a' -> 1 bits, 'b' -> 2 bits
- ◉ 'a' -> 2 bits, 'b' -> 2 bits

- ◯ 'a' -> 2 bits, 'b' -> 3 bits
- ◯ 'a' -> 3 bits, 'b' -> 3 bits
- ◯ 'a' -> 3 bits, 'b' -> 4 bits

4

Suppose you are compressing a document with N total characters and M unique characters. How many nodes will there be in the Huffman coding tree? *

- ◯ O(N)
- ◉ O(M)
- ◯ O(N + M)
- ◯ O(N log(N))
- ◯ O(M log(M))
- ◯ O(N^2)
- ◯ O(M^2)

java.util.PriorityQueue Class

- Kept in sorted order, smallest out first
  - Objects must be Comparable OR provide Comparator to priority queue

```
PriorityQueue<String> pq = new PriorityQueue<>();
pq.add("is");
pq.add("Compsci 201");
pq.add("wonderful");
while (! pq.isEmpty()) {
    System.out.println(pq.remove());
}
```

```
Compsci 201
is
wonderful
```

```
PriorityQueue<String> pq = new PriorityQueue<>(
        Comparator.comparing(String::length));
pq.add("is");
pq.add("Compsci 201");
pq.add("wonderful");
while (! pq.isEmpty()) {
    System.out.println(pq.remove());
}
```

```
is
wonderful
Compsci 201
```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java.util PriorityQueue basic methods

Method	Behavior	Runtime Complexity
`add(element)`	Add an element to the priority queue	O(log(N)) comparisons
`remove()`	Remove and return the minimal element	O(log(N)) comparisons
`peek()`	Return (do *not* remove) the minimal element	O(1)
`size()`	Return number of elements	O(1)

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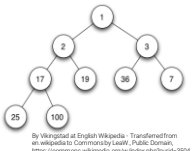
Binary Heap at a high level

A **binary heap** is a binary tree satisfying the following structural invariants:

- ***heap property***: *every node is less than or equal to its successors*, and
- ***shape property***: *the tree is* ***complete*** (full except possibly last level, in which case it should be filled from left to right)



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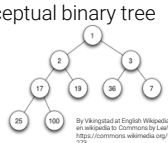
How are binary heaps typically implemented?

- Normally think about a conceptual binary tree underlying the binary heap.



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- Usually implement with an array
  - minimizes storage (no explicit points/nodes)
  - simpler to code, no explicit tree traversal
  - faster too (constant factor, not asymptotically)---children are located by index/position in array

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Aside: How much less memory?

- Storing an int takes 4 bytes = 32 bits on most machines.
- Storing one *reference to an object* (a memory location) takes 8 bytes = 64 bits on most machines.

- For a heap storing N integers…
  - Array of N integers takes ~ 4N bytes.
  - Binary tree where each node has an int, left, and right reference takes ~20N bytes.
  - So maybe a 5x savings in memory (just an estimate). ***Not*** an asymptotic improvement.

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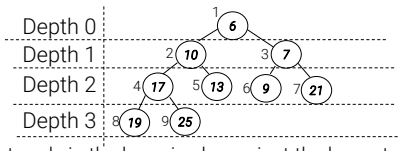
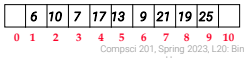
Using an array for a Heap

- Makes it easy to keep track of last "node" in "tree"
- Index positions in the tree level by level, left to right:



- Last node in the heap is always just the largest non-empty index
- Can use indices to represent as an array!



(ArrayList if you want it to be growable)

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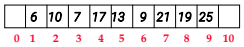
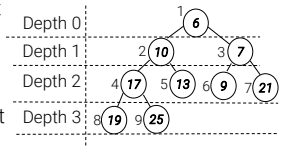
Properties of the Heap Array

- Store "node values" in array beginning at index 1
  - Could 0-index, Zybook does this
- Last "node" is always at the max index
- Minimum "node" is always at index 1
- **peek** is easy, return first value.
  - How about add?
  - Remove?





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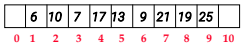
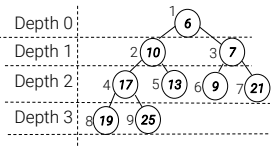
Relating Nodes in Heap Array

- When 1-indexing: For node with index k
  - left child: index 2*k
  - right child: index 2*k+1
  - parent: index k/2
- Why? Follows from:
  - Heap is *complete*, and
  - Complete binary tree has 2^d nodes at depth d (except last level)



Integer division



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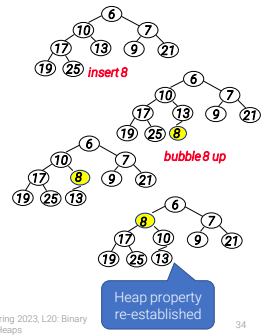
Adding values to heap in pictures

- Add to first open position in last level of the tree
  - (really, add to end of array)
- Shape property satisfied, but not heap property
- Fix it: Swap with parent if heap property violated
  - Stop when parent is smaller,
  - or you reach the root



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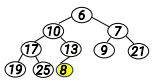
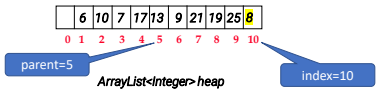
Heap add implementation



```
24  public void add(Integer value) {
25      heap.add(value); // add to last position
26      size++;
27
28      int index = size; // note we are 1-indexing
29      int parent = index / 2;
30
31      while(parent >= 1 && heap.get(parent) > heap.get(index)) {
32          swap(index, parent);
33          index = parent;
34          parent /= 2;
35      }
36  }
```



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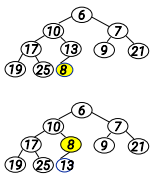
Heap add implementation



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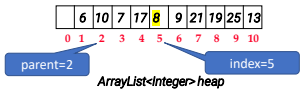

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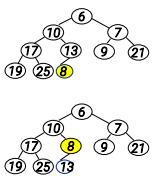
Heap add implementation



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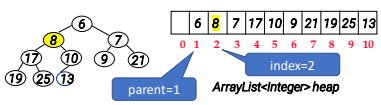

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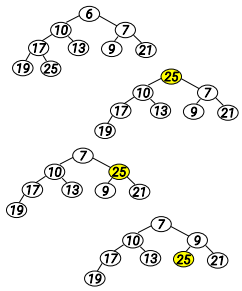
Heap remove in pictures

- Always return root value
- How to repair shape into a single tree?
  - Replace root with last node in the heap
  - While heap property violated, swap with ***smaller*** child.



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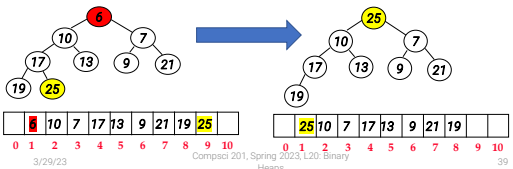
Heap remove implementation

```
38  public Integer remove() {
39      if (size < 1) { return null; }
40      Integer retVal = heap.get(index:1);
41      heap.set(index:1, heap.get(size));
42      heap.remove(size);
43      size--;
44      if (size == 0) { return retVal; }
```

Get the minimal value

Replace "root" with "last node"

Delete "last node"



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Heap remove implementation

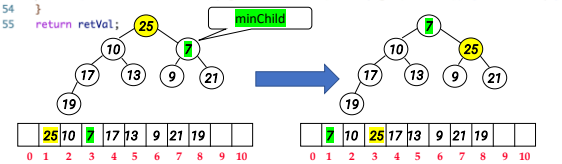
```
46  int index = 1;
47  int minChild = 2;
48  if (size > 2 && heap.get(index:3) < heap.get(index:2)) { minChild = 3; }
49  while (minChild <= size && heap.get(index) > heap.get(minChild)) {
50      swap(index, minChild);
51      index = minChild;
52      minChild = minChild * 2;
53      if (size > minChild && heap.get(minChild + 1) < heap.get(minChild)) { minChild++; }
54  }
55  return retVal;
```

Find the smaller of 2 child nodes

Swap

Violating heap property

minChild



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Heap remove implementation

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48  if (size > 2 && heap.get(index:3) < heap.get(index:2)) { minChild = 3; }
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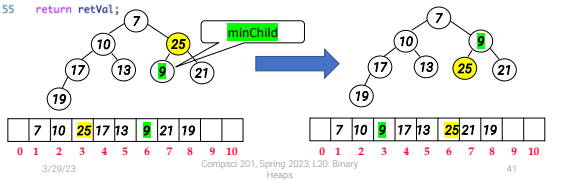
50      swap(index, minChild);
51      index = minChild;
52      minChild = minChild * 2;
53      if (size > minChild && heap.get(minChild + 1) < heap.get(minChild)) { minChild++; }
54  }
55  return retVal;
```

Update minChild



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Heap remove implementation

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48  if (size > 2 && heap.get(index:3) < heap.get(index:2)) { minChild = 3; }
49  while (minChild <= size && heap.get(index) > heap.get(minChild)) {
50      swap(index, minChild);
51      index = minChild;
52      minChild = minChild * 2;
53      if (size > minChild && heap.get(minChild + 1) < heap.get(minChild)) { minChild++; }
54  }
55  return retVal;
```
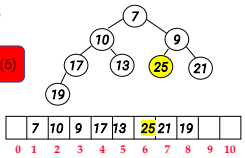

2*6 = 12 > size

Return retVal (6)



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Heap Complexity

- Claimed that:
  - Peek: O(1)
  - Add: O(log(N))
  - Remove: O(log(N))
- On a heap with N values. Why?
  - Peek: Easy, return first value in an Array
  - Complete binary tree always has height O(log(N)).
  - .add and remove "traverse" ***one*** root-leaf path, length at most O(log(N)).

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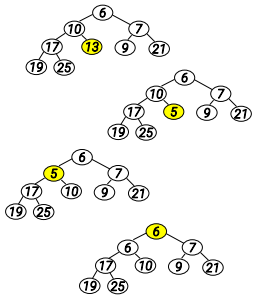
decreaseKey Operation?

- Suppose we decrease the 13 to 5.
- Violates heap property
- Fix like in the add operation:
  - While violating heap property, swap with parent



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decreaseKey NOT in java.util

- decreaseKey is important for some algorithms, but not supported in many standard libraries (including the java.util.PriorityQueue)
- Why not?
  - Note that binary heap does not support efficient *search*
  - In order to do decreaseKey in O(log(n)) time, need to store *references/indices* of all the "nodes."
  - Adds overhead, not done in java.util

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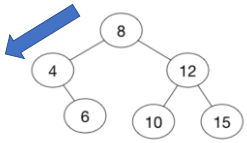
Alternative Implementation: Binary Search Tree

- If your keys happen to be unique…
- Can support O(log(n)) add & remove (smallest) using a binary search tree!
- Smallest is leftmost child



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PriorityQueue (with unique keys) using a java.util TreeSet

```
import java.util.TreeSet;

public class BSTPQ<T extends Comparable<T>> {
    private TreeSet<T> bst;

    public BSTPQ() { bst = new TreeSet<>(); }
    public void add(T element) { bst.add(element); }
    public int size() { return bst.size(); }
    public T peek() { return bst.first(); }

    public T remove() {
        T returnValue = bst.first();
        bst.remove(returnValue);
        return returnValue;
    }

    public void decreaseKey(T oldKey, T newKey) {
        bst.remove(oldKey);
        bst.add(newKey);
    }
}
```

`first` gives smallest element in TreeSet in O(log(n)) time

Can `decreaseKey` by removing and then re-adding, both O(log(n)) time for a TreeSet

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Disadvantages to using a Binary Search Tree for your priority queue?

1. All elements must be unique
2. Not array-based, uses more memory and has higher constant factors on runtime
3. Much harder to implement with guarantees that the tree will be balanced.

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