CompSci 370
Uninformed Search

Ron Parr
Department of Computer Science
Duke University

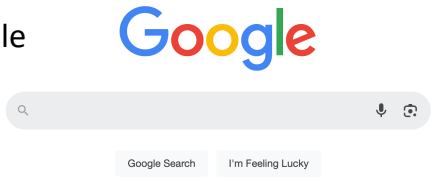
With thanks to Vince Conitzer for some slides and figures and thanks to Kris Hauser for many slides

What is Search?

- Search is a basic problem-solving method
  - We start in an initial state
  - We examine states that are (usually) connected by a sequence of actions to the initial state
- Note: Search is (usually) a thought experiment (separate topic: Real Time Search)
- We aim to find a solution, which is a sequence of actions that brings us from the initial state to the goal state, *possibly* minimizing cost

Search vs. Web Search

- When we issue a search query using Google, does Google really go poking around the web for us?



- Not in real time!
- Google spiders the web continually, caches results
- Uses page rank algorithm (a simple eigenvector problem) to find the most "popular" web pages that are consistent with your query
- Modern Google search also uses more advanced AI methods

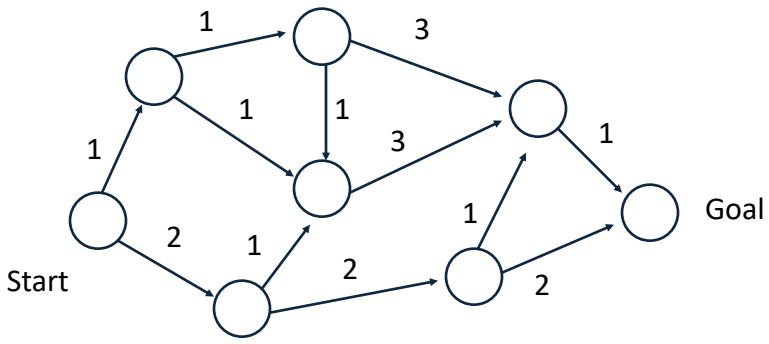
Overview

- Problem Formulation
- Uninformed Search – constant cost
  - DFS, BFS, IDDFS, etc.
- Non-constant cost

Problem Formulation

- Components of a search problem
  - State space & initial state
  - Actions
  - Goal Test
  - Edge costs (cost of moving from one state to another upon taking an action)
    - May be constant or varying per edge (initially we assume constant)
    - Assumed to be > 0
- Optimal solution = lowest path cost to goal

Example: Path Planning, e.g. Google Maps

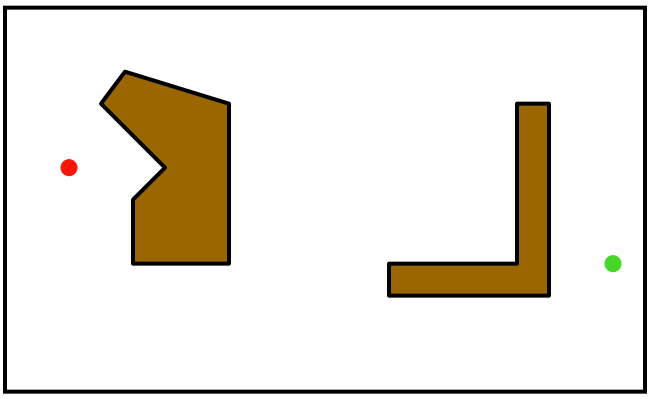


Find shortest source to destination using available roads

Other Search Problems

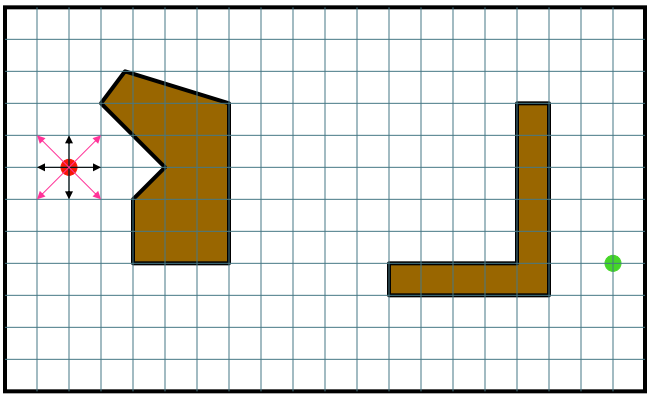
- Drug design
- Logistics
  - Route planning
  - Tour Planning
- Assembly sequencing
- Internet routing
- Robot motion/path planning

Robot Path Planning



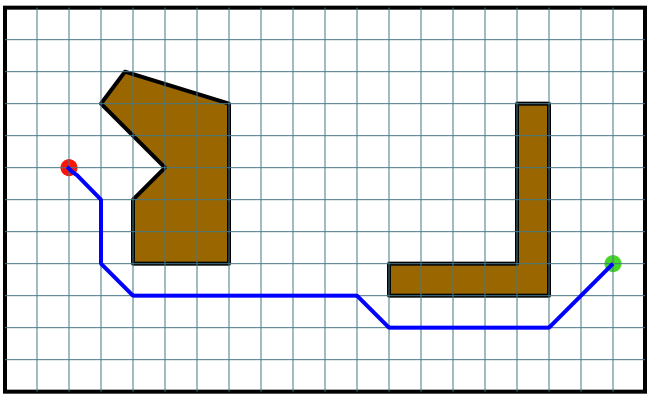
What is the state space?

Formulation #1



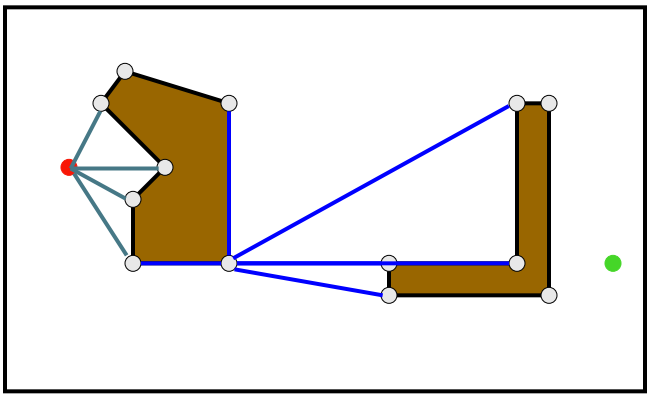
Cost of one horizontal/vertical step = 1
Cost of one diagonal step = sqrt(2)

Optimal Discretized Solution



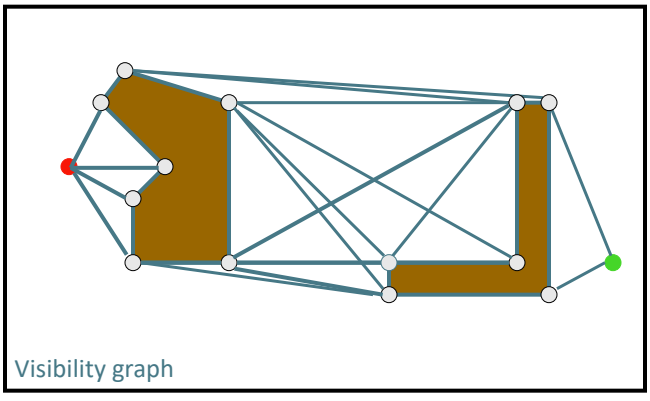
This path is the shortest in the discretized state space, but not in the original continuous space

Formulation #2



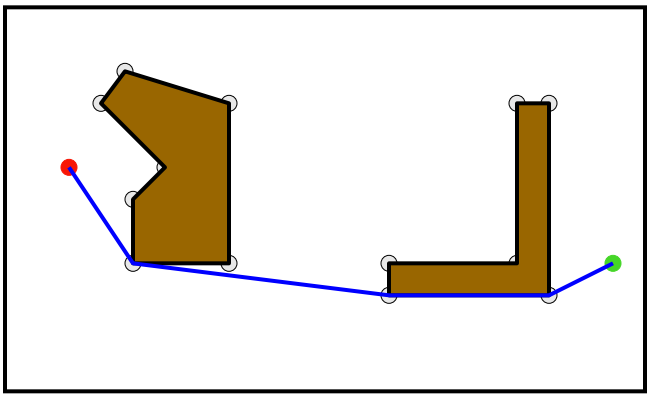
Cost of one step: length of segment

Formulation #2



Cost of one step: length of segment

Solution Path



The shortest path in this state space is also the shortest in the original continuous space

13

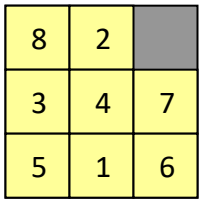
Take Home Points

- States = modeling choice about the world
- Trade offs often exist:
  - Example 1: Discretization is easy to work with, but optimal solution to may be suboptimal in the real world
  - Example 2: More clever representations may require ingenuity to discover, or use, but may have benefits in real world
- Always keep modeling and solving distinct in your head

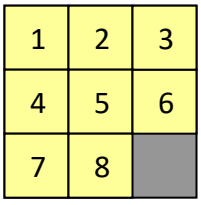
Basic Search Concepts (with more precision/detail)

- Search tree: Internal representation of our progress
- Nodes: Places in search tree (states exist in the problem space)
- Actions: Connect states to next states (nodes to nodes)
- Expansion: Generation of next states (nodes)
- Arc cost: Cost of moving from one state to another
- Frontier: Set of nodes visited, but not expanded
- Branching factor: Max no. of successors = b
- Goal depth: Depth of *shallowest* goal = d (root is depth 0, possibility of multiple goal states!)

Example: 8-Puzzle



Initial state

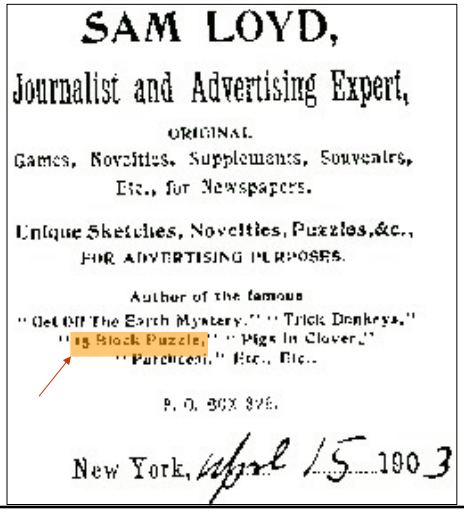


Goal state

State: Arrangement of 8 numbered tiles & empty tile on a 3x3 board

15-Puzzle

- Introduced (?) in 1878 by Sam Loyd, who dubbed himself "America's greatest puzzle-expert"



SAM LOYD,

Journalist and Advertising Expert,

ORIGINAL
Games, Novelties, Supplements, Souvenirs,
Etc., for Newspapers.

Unique Sketches, Novelties, Puzzles, &c.,
FOR ADVERTISING PURPOSES.

Author of the famous
"Get Off The Earth Mystery," "Trick Donkeys,"
"[illegible]g Block Puzzle," "Pigs in Clover,"
"Parchesi," Etc., Etc.

P. O. BOX 876.

New York, April 15 1903

15-Puzzle

- Sam Loyd offered $1,000 of his own money to the first person who would solve the following problem:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

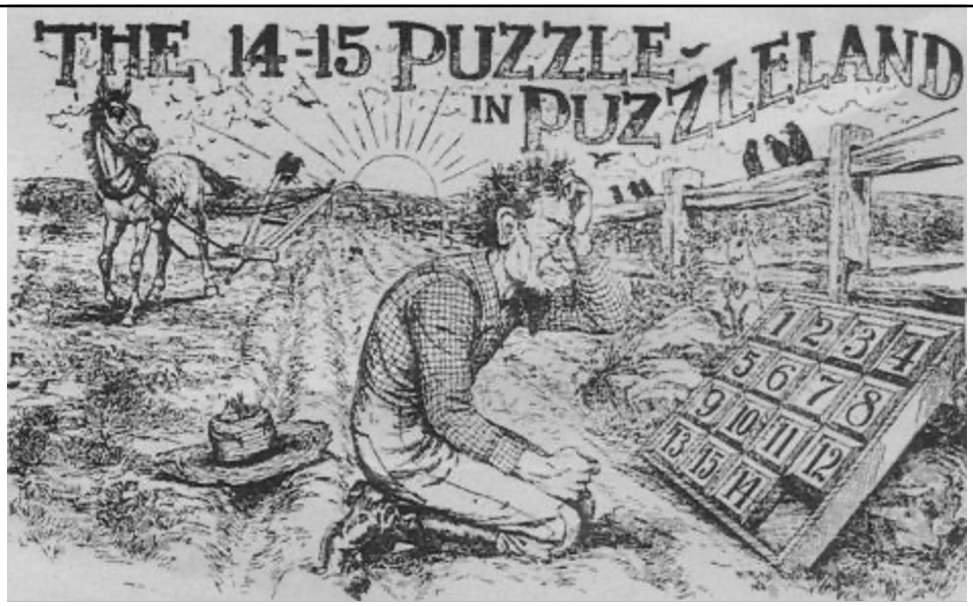
?

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

How big is the state space of the (n^2-1)-puzzle?

- 8-puzzle → 9! = 362,880 states
- 15-puzzle → 16! ~ 2.09 x 10^{13} states
- 24-puzzle → 25! ~ 10^{25} states

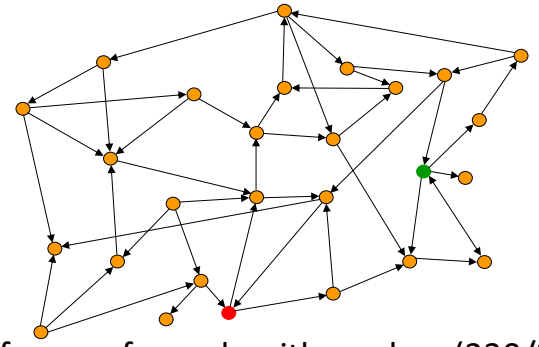
- But only half of these states are reachable from any given state (and you may not know that in advance)



No one ever won the prize !!

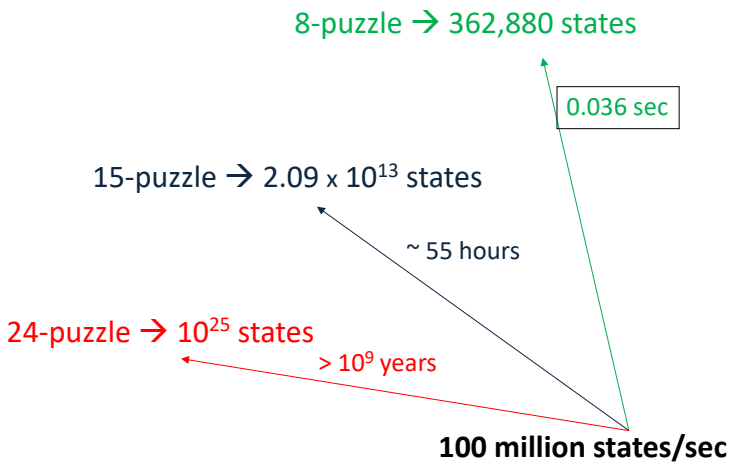
Searching the State Space

- Often infeasible (or too expensive) to build complete representation of the state graph



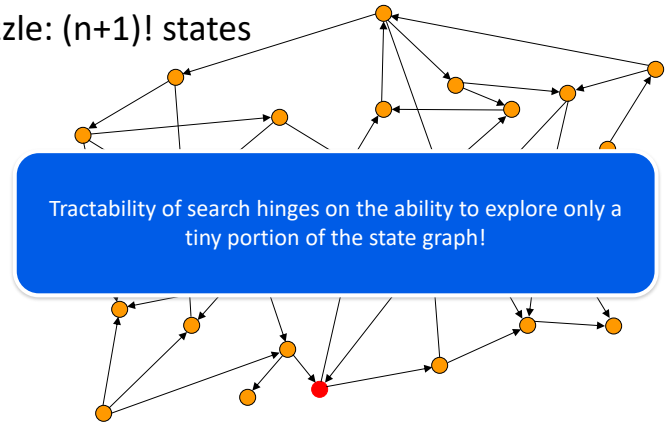
- Key difference from algorithms class (230/330), where it is typically assumed that graph fits in memory

8-, 15-, 24-Puzzles

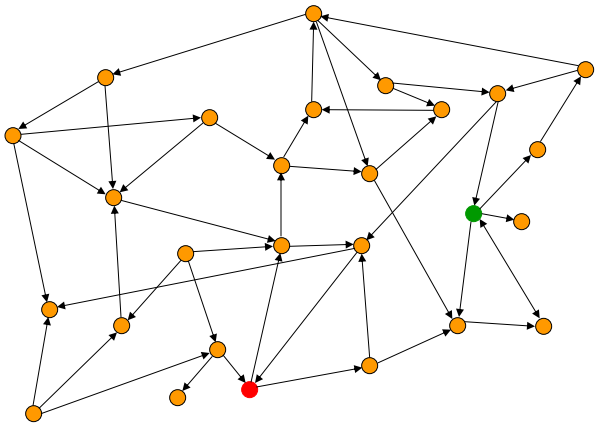


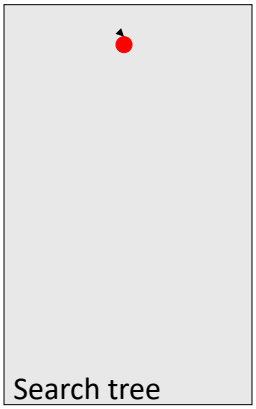
Intractability

- Constructing the full state graph is intractable for many interesting problems
- n-puzzle: (n+1)! states

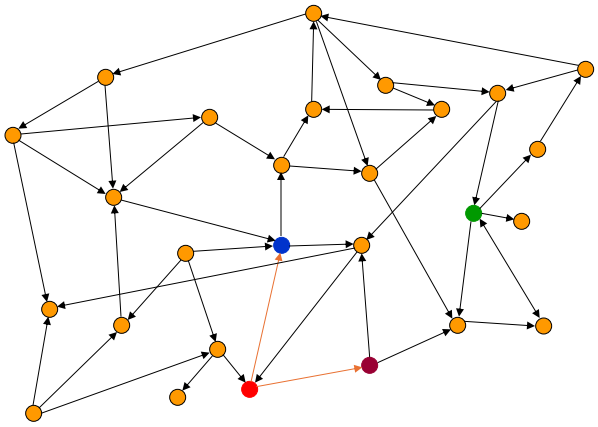


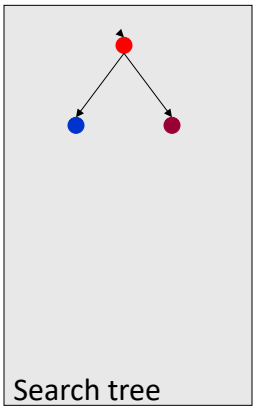
Searching the State Space



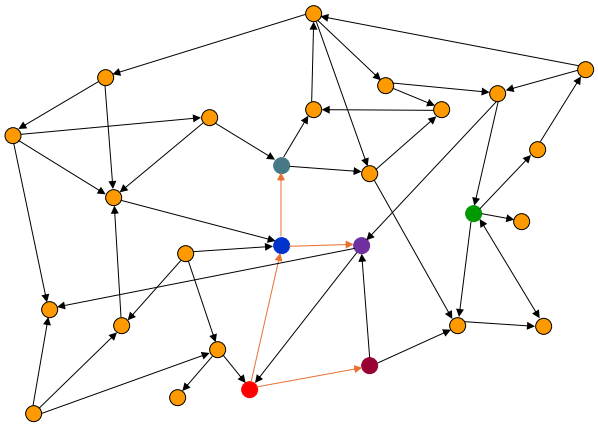


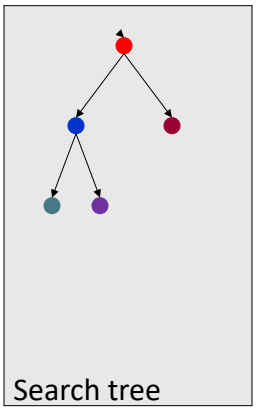
Searching the State Space



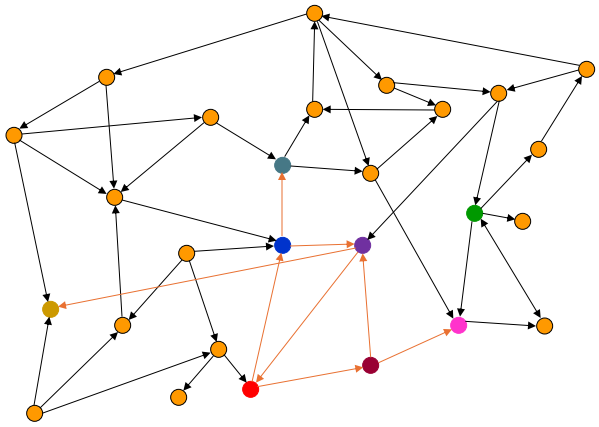


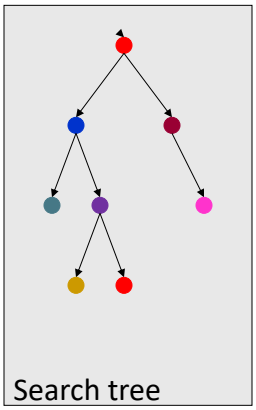
Searching the State Space



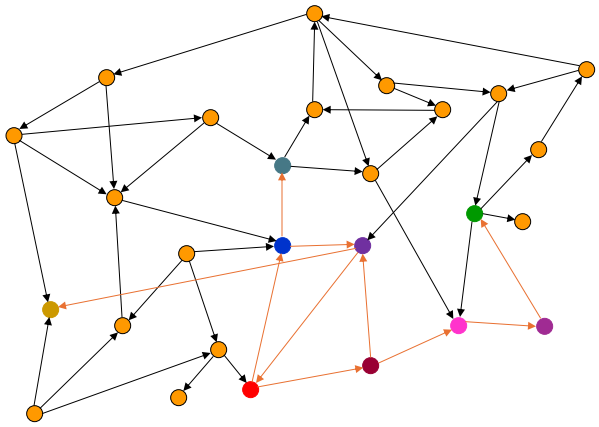


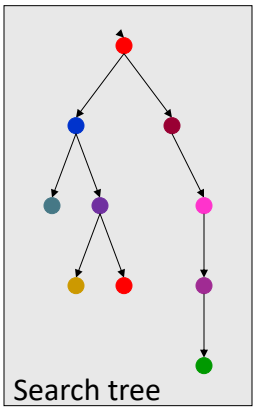
Searching the State Space



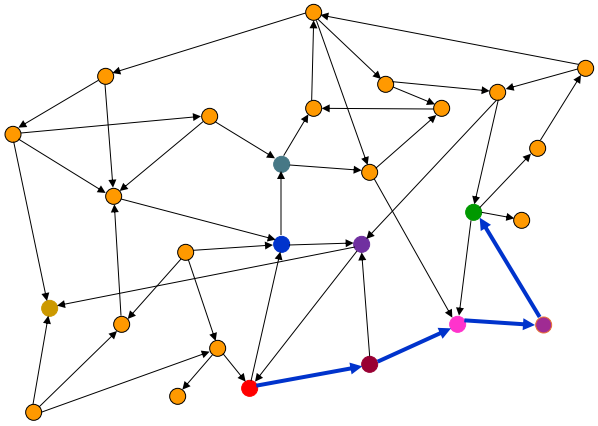


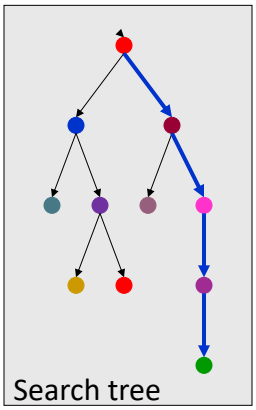
Searching the State Space





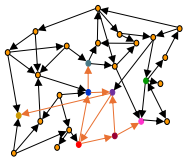
Searching the State Space



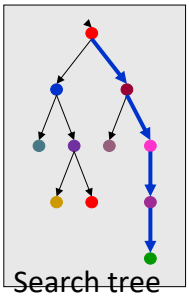


Keeping things clear in your mind

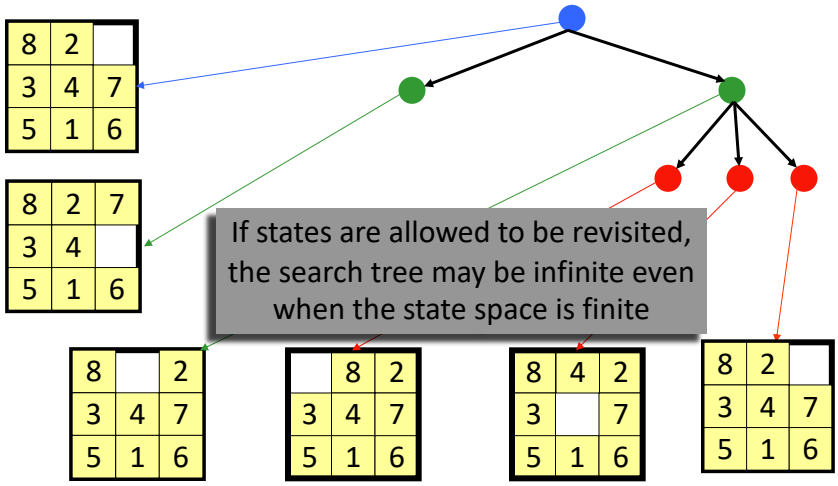
- State space:
  - Can always be represented with a graph showing connections between states
  - State space may also be a tree
  - Nodes correspond to states (one to one)



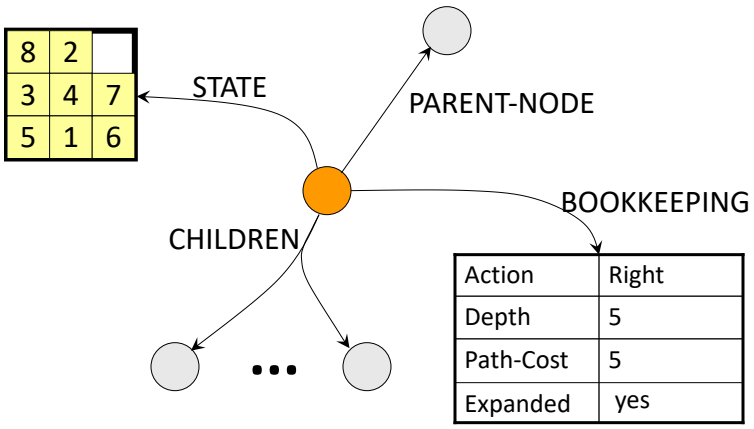
- Search tree:
  - Always a tree
  - Nodes in search tree (search nodes) point to states
  - Same state could appear at multiple nodes in the search tree (but we try to avoid that)



Search Nodes and States (8-puzzle example)



Data Structure of a Node



Note: This slide is application/language/ implementation agnostic. Not all potential attributes of a node are needed in all cases.

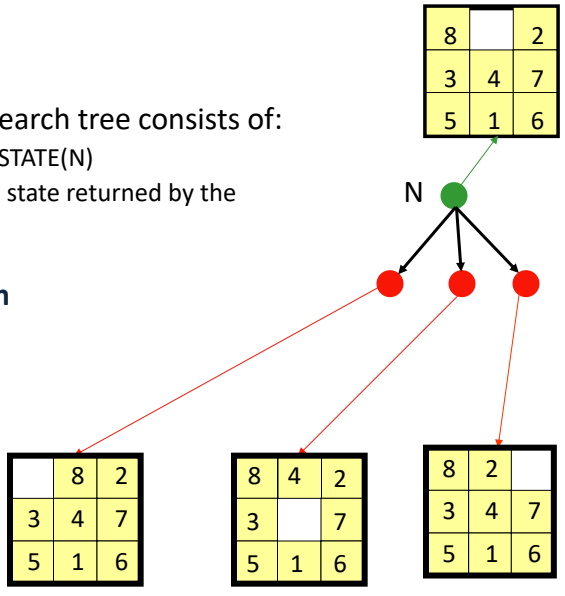
Depth of a node N = length of path from root to N (depth of the root = 0)

Implementation

- Implementation of node data structure depends upon
  - Implementation
  - Programming language
  - Information expected by user
- Example 1: If all you care about is minimizing the number of steps, there is no need to store cost (since cost = depth)
- Example 2: Can return path to goal (sequence of states) associated with solution by storing path to reach a node as part of the node data structure
- Example 3: Can avoid storing path explicitly, but reconstruct it using links from nodes to their parents.

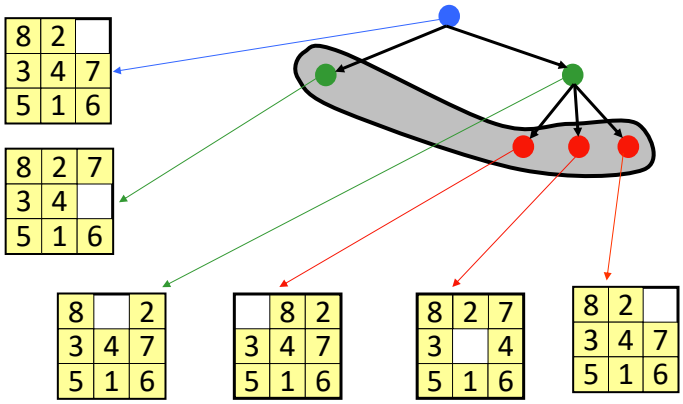
Node expansion

- The expansion of a node N of the search tree consists of:
  - Evaluating the successor function on STATE(N)
  - Generating a child node of N for each state returned by the successor function
- **node generation ≠ node expansion**



Frontier of Search Tree

The frontier is the set of all search nodes that haven't been expanded yet



Search Strategy

- The frontier is the set of all search nodes that haven't been expanded yet
- Implemented as a (priority) queue FRONTIER
  - INSERT(node, FRONTIER)
  - REMOVE(FRONTIER)
- The ordering of the nodes in FRONTIER defines the search strategy

Generic Tree Search (***assumes*** state space is a tree)

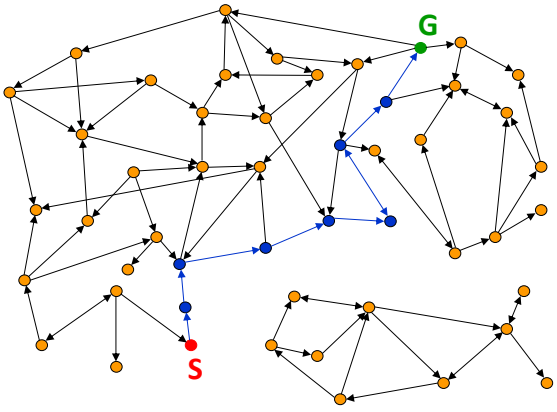
TREE-SEARCH(initial-state)

1. If GOAL?(initial-state) then return initial-state
2. INSERT(initial-node, FRONTIER)
3. Repeat:
4. If empty(FRONTIER) then return failure
5. N ← REMOVE(FRONTIER)
6. s ← STATE(N)
7. For every state s' in SUCCESSORS(s)
8. Create a new node N' as a child of N
9. If GOAL?(s') then return path or goal state
10. INSERT(N', FRONTIER)

Expansion of N

Solution to the Search Problem

- A solution is a path connecting the initial node to a goal node (any goal)
- The cost of a path is the sum of the arc costs along this path
- An optimal solution is a solution path of minimum cost
- There might be no solution !



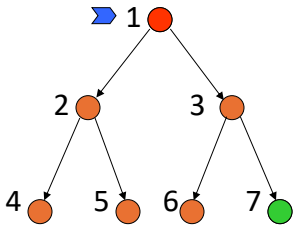
Recall: Typically assume costs > 0

Algorithm Performance Measures

- Completeness:
  - Does it find a solution when one exists?
- Optimality:
  - Does it return a min cost path whenever solution exists?
- Complexity (space or time):
  - Resources required by the algorithm

Breadth-First Search

- FRONTIER is a FIFO Queue

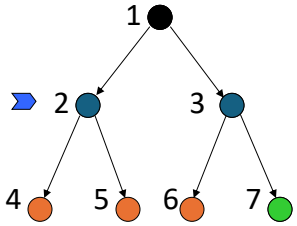


FRONTIER = (1)

Note: Typically assume that ties broken in left-to right order.

Breadth-First Search

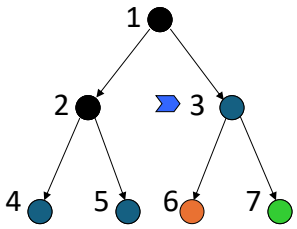
- FRONTIER is a FIFO Queue



FRONTIER = (2, 3)

Breadth-First Search

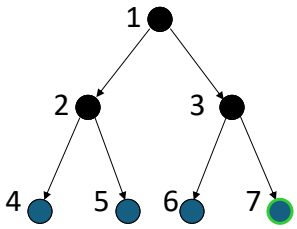
- FRONTIER is a FIFO Queue



FRONTIER = (3, 4, 5)

Breadth-First Search

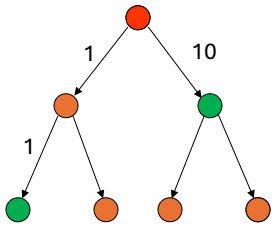
- FRONTIER is a FIFO Queue



FRONTIER = (4, 5, 6, 7)

BFS Properties

- Assume
  - Branching factor: b
  - Depth of shallowest goal: d
- Completeness: Y
- Optimality: (Y for constant cost, N for variable cost)
- Time complexity (nodes generated): $O(b^d)$
- Space complexity: $O(b^d)$



How bad is exponential in d?

d	# Nodes	Time	Memory
2	111	.01 msec	11 Kbytes
4	11,111	1 msec	1 Mbyte
6	~10^6	1 sec	100 Mb
8	~10^8	100 sec	10 Gbytes
10	~10^{10}	2.8 hours	1 Tbyte
12	~10^{12}	11.6 days	100 Tbytes
14	~10^{14}	3.2 years	10,000 Tbytes

Assumptions: b = 10; 1,000,000 nodes/sec; 100bytes/node

Bi-directional BFS

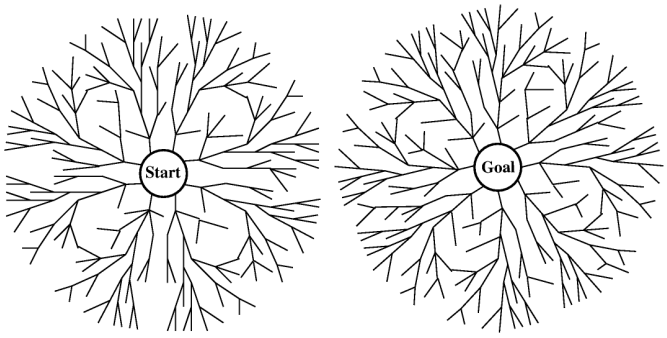


image from cs-alb-pc3.massey.ac.nz/notes/59302/fig03.17.gif

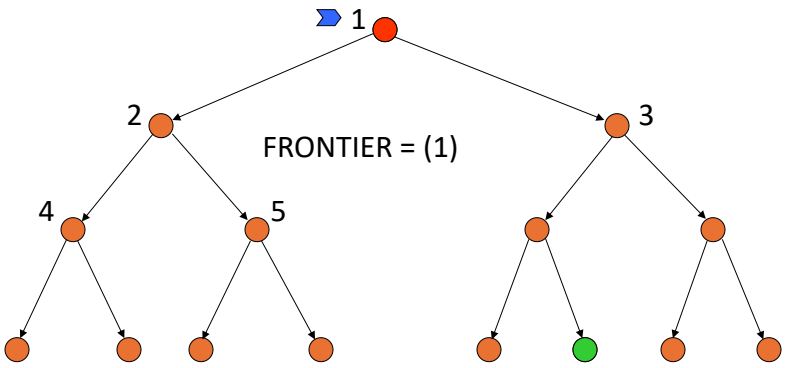
$$b^{d/2} + b^{d/2} << b^d$$

Issues with Bi-directional BFS

- Uniqueness of goal
  - Suppose goal is parking your car at airport
  - Huge no. of possible goal states
    - Configurations of other vehicles
    - Which space you use
- Invertability of actions (can the successor function be inverted easily?)

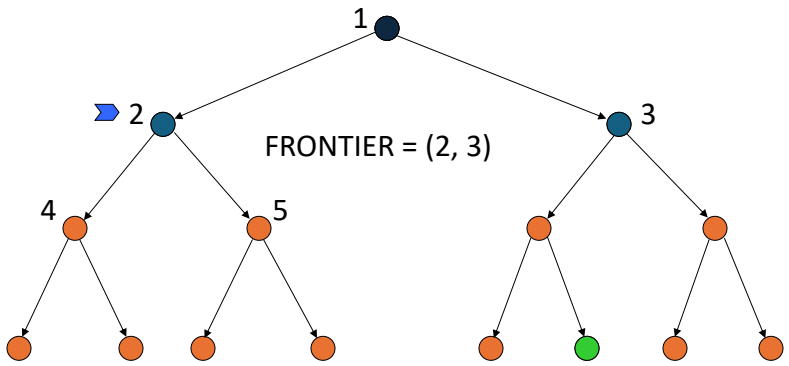
Depth-First Search

- FRONTIER is a LIFO Queue



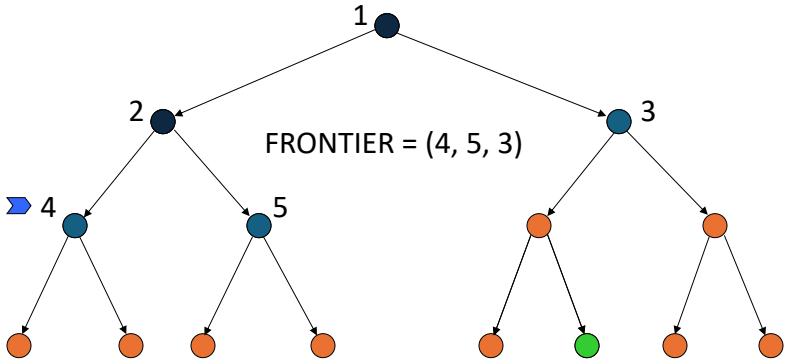
Depth-First Search

- FRONTIER is a LIFO Queue



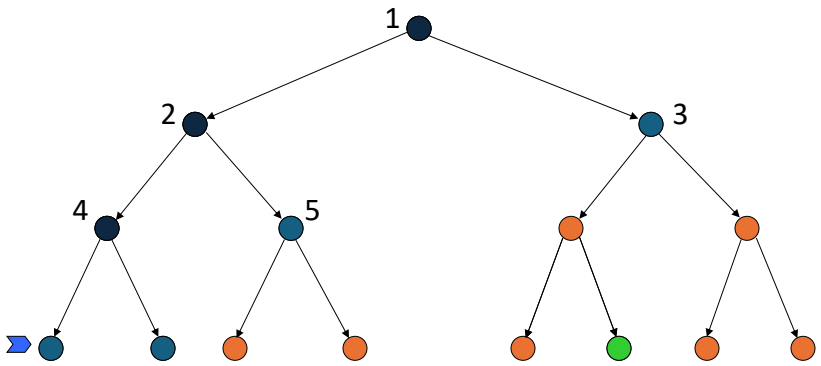
Depth-First Search

- FRONTIER is a LIFO Queue



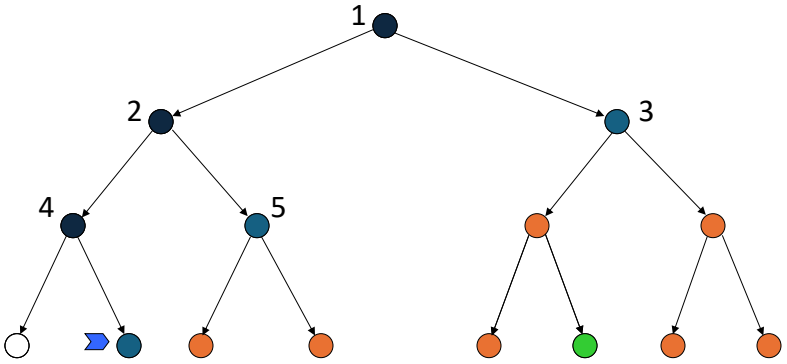
Depth-First Search

- FRONTIER is a LIFO Queue



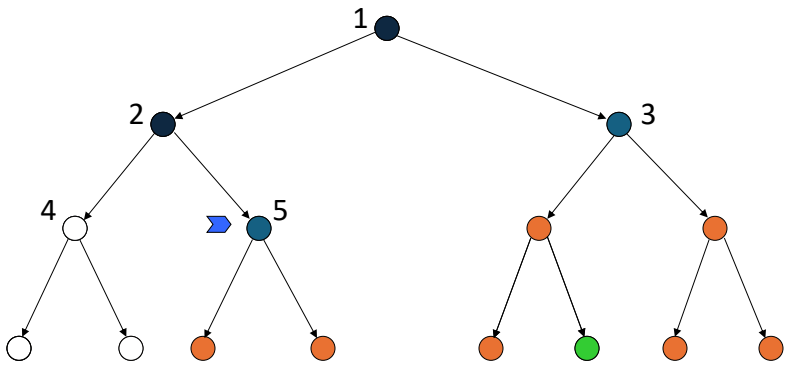
Depth-First Search

- FRONTIER is a LIFO Queue



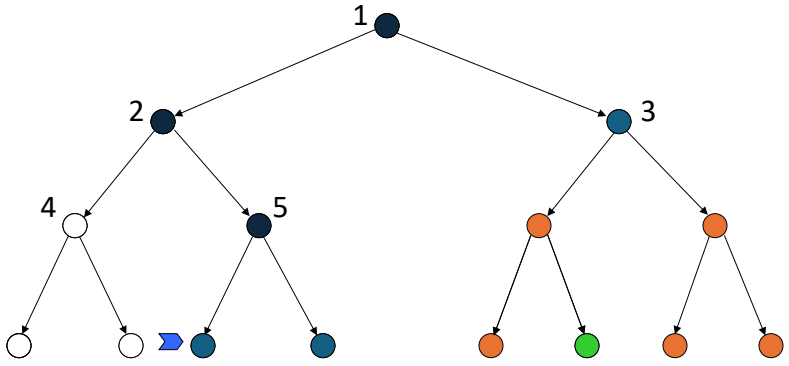
Depth-First Search

- FRONTIER is a LIFO Queue



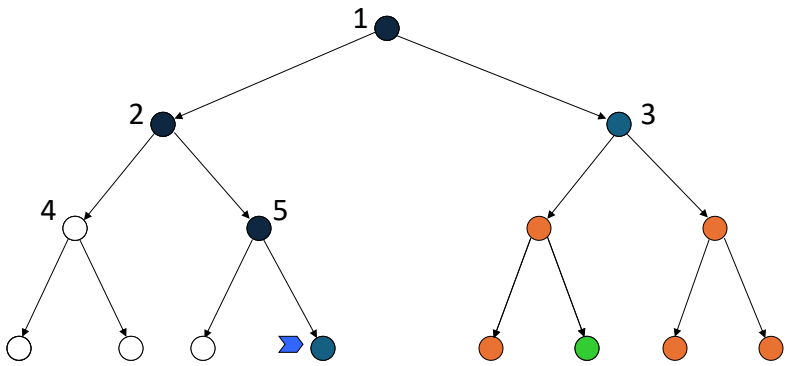
Depth-First Search

- FRONTIER is a LIFO Queue



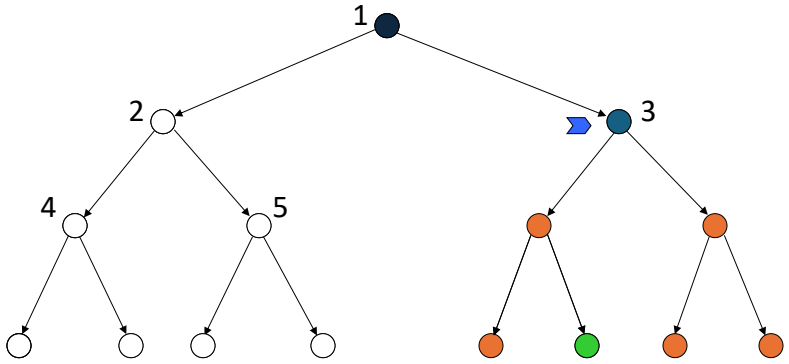
Depth-First Search

- FRONTIER is a LIFO Queue



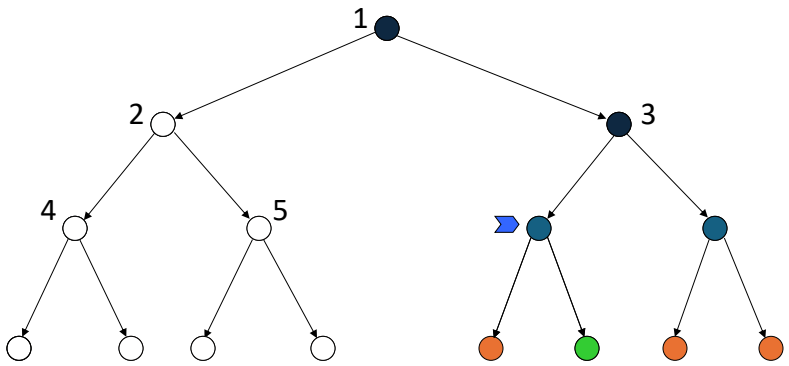
Depth-First Search

- FRONTIER is a LIFO Queue



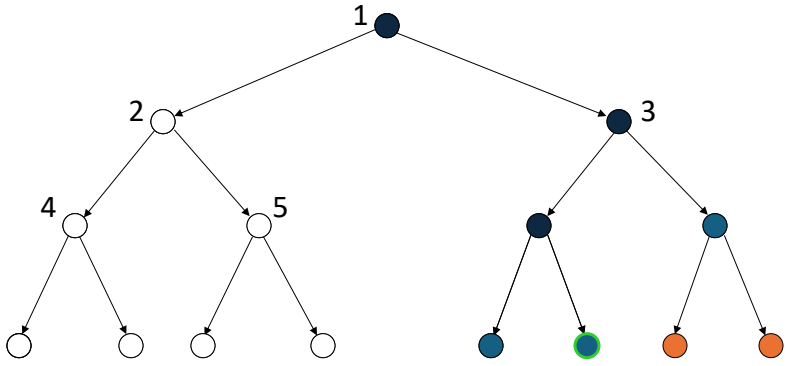
Depth-First Search

- FRONTIER is a LIFO Queue



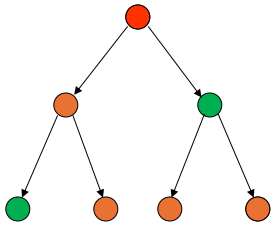
Depth-First Search

- FRONTIER is a LIFO Queue



DFS Properties

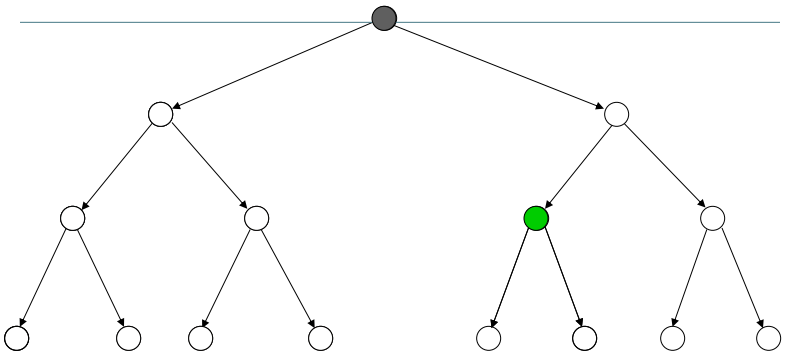
- Completeness: (Y for finite trees, N for infinite trees)
- Optimality: N (Can you think of an example?)
- Time complexity: $O(b^m)$ (m = depth we hit, m>d?)
- Space complexity: O(bm) (bounded for trees)



Iterative Deepening (IDS or IDDFS)

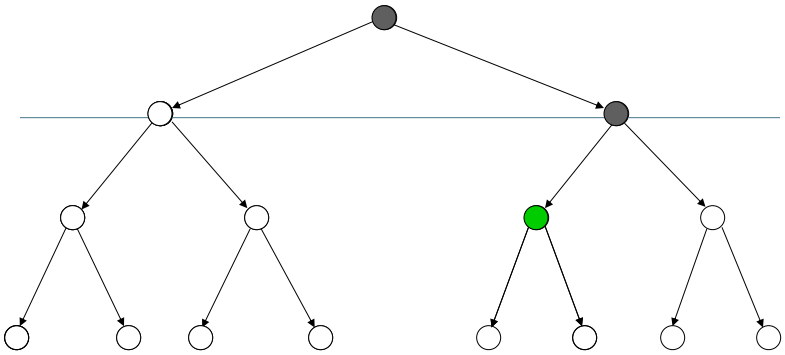
- Want:
  - DFS memory requirements
  - BFS optimality, completeness
- Idea:
  - Do a depth-limited DFS for depth m
  - Iterate over m

Iterative Deepening

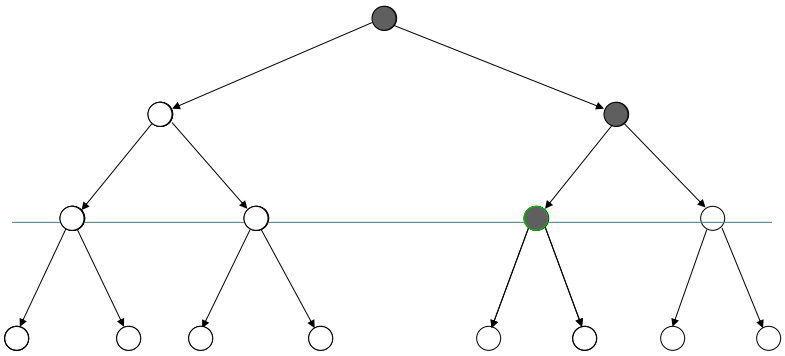


Note: The IDDFS slides are animated, showing DFS running down to the red line on each slide.

Iterative Deepening



Iterative Deepening



IDDFS Properties

- Completeness: Y
- Optimality: (whenever BFS is optimal)
- Time complexity: $O(b^{d+1})$
- Space complexity: O(bd)

IDDFS vs. BFS

Theorem: IDDFS generates no more than twice as many nodes for a binary tree as BFS.

Proof: Assume the tree bottoms out at depth d, BFS generates: $BFS(d) = 2^{d+1} - 1$

In the worst case, IDDFS does no more than:

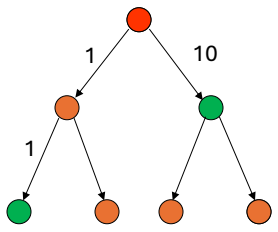
$$\sum_{i=0}^{d}(2^{i+1}-1) = 2\sum_{i=0}^{d}2^{i} - \sum_{i=0}^{d}1 = 2(2^{d+1}-1) - (d+1) < 2(2^{d+1}-1) < 2\times BFS(d)$$

What about b-ary trees?

IDDFS relative cost is lower!

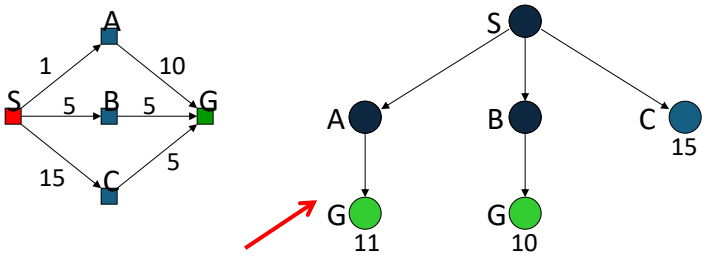
Non-constant Costs

- Arcs between states can have variable costs
- The cost of the path to each node N is g(N) = Σ costs of arcs from root to N
- Breadth-first is no longer optimal with variable arc costs!



Uniform-Cost Search (UCS)

- Expand node in FRONTIER with the cheapest path so far
- Frontier is a priority queue prioritized on g(N)
- Needs one more fix to be optimal



Suboptimal path!

(how to fix this?)

Search Algorithm #2

TREE-SEARCH2(initial-state)

1. If GOAL?(initial-state) then return initial-state
2. INSERT(initial-node,FRONTIER)
3. Repeat:
4. If empty(FRONTIER) then return failure
5. N ← REMOVE(FRONTIER)
6. s ← STATE(N)
7. If GOAL?(s) then return path or goal state
8. For every state s' in SUCCESSORS(s)
9. Create a new node N' as a child of N
10. INSERT(N',FRONTIER)

The goal test is applied to a node when this node is **expanded**, not when it is generated.

Now, UCS is optimal!

Avoiding Revisited States

- Requires comparing state descriptions
- Applied to breadth-first search:
  - Store all states associated with generated nodes in VISITED
  - If the state of a new node is in VISITED, then discard the node

Implemented as hash-table (e.g. python dictionary)
or as some other explicit data structure

Avoiding Revisited States in DFS

- Depth-first search:
  - Solution 1 (similar to BFS approach):
    - Store all states in current path in VISITED
    - If the state of a new node is in VISITED, then discard the node
    - Only avoids loops
  - Solution 2:
    - Store **all generated states** in VISITED
    - If the state of a new node is in VISITED, then discard the node
    - Avoids ever revisiting the same state twice
    - Same space complexity as breadth-first !

Avoiding Revisited States in UCS

- UCS property: For any state S, when the first node N such that STATE(N) = S is expanded, the path to N is the best path from the initial state to S (i.e., true for all states, not just goal state)

- So:
  - When a node is **expanded**, store its state into VISITED
  - When a new node N is generated:
    - If STATE(N) is in VISITED, discard N
    - If there exists a node N' in the frontier such that STATE(N') = STATE(N), discard the node -- N or N' – w/highest cost (i.e., it's possible we found a shortcut to a node already in the frontier, but not yet expanded)

Search Algorithm #3

GRAPH-SEARCH(initial-state)

1. If GOAL?(initial-state) then return initial-state
2. INSERT(initial-node,FRONTIER)
3. Repeat:
4. If empty(FRONTIER) then return failure
5. N ← REMOVE(FRONTIER)
6. s ← STATE(N)
7. Add s to VISITED
7. If GOAL?(s) then return path and/or goal state
8. For every state s' in SUCCESSORS(s)
9. Create a new node N' as a child of N
10. If s' is in VISITED then discard N'
11. else if there is N'' in FRONTIER with STATE(N')=STATE(N'') and g(N'') <= g(N') then discard N'
12. else INSERT(N',FRONTIER)

Uninformed Search Summary

- Many variations on same basic algorithm
- Key differences:
  - How frontier is implemented (FIFO, LIFO, priority queue)
  - When goal test is applied
  - Whether and how assiduously visited list is maintained
- Big impact on:
  - Completeness
  - Optimality
  - Complexity