

Due: Thursday, January 15
35 points

On homework, you may discuss with other students in the course about how to solve a problem, but the write-up should be your own. You **must include the names** of any students you consulted with. Give credit where credit is due.

1. (5 pts) Given the sets of integers below, describe the new sets created using similar notation.

$$S_1 = \{0, 4, 8, 12, \dots\}$$

$$S_2 = \{n > 0 \mid n \text{ is divisible by } 5\}$$

$$S_3 = \{n > 0 \mid n \text{ is even}\}$$

$$S_4 = \{3, 5\}$$

$$S_5 = \{3, 4, 5, 6, 7, 8\}$$

(a) $S_1 \cap S_2 =$

(b) $S_3 \cap S_4 =$

(c) $S_1 \cup S_3 =$

(d) $S_5 - S_4 =$

(e) $S_3 \times S_4 =$

2. (3 pts) True or False?

(a) $\emptyset \subseteq \{x, y, \{x, y\}\}$

(b) $\{x, y\} \subseteq \{x, y, \{x, y\}\}$

(c) $\{x\} \in \{x, y, \{x, y\}\}$

3. (4 pts) Prove by induction $1 + 3 + 5 + 7 + \dots + (2n - 1) = n^2$ for all $n > 0$. Show all steps (basis, IH, and IS).

4. (4 pts) Prove by induction $2^n < n!$, $n \geq 4$

5. (2 pts) Explain what is wrong with the following proof by induction that all Duke computer science majors are the same gender.

Proof by induction on the number of Duke computer science majors.

Basis: There is only one Duke CPS major. Clearly all such majors are the same gender.

I.H.: In any group of N Duke CPS majors, the students are the same gender.

I.S.: Consider a group of $N+1$ CPS majors. Remove one student. The remaining group has N students. By the induction hypothesis, the N students are the same gender. Put the student back and remove a different student, also a group of N students. Also by the I.H., these N students are the same gender. Since the student not currently in the group was shown to have the same gender one step earlier, all $N+1$ students have the same gender. Thus all CPS majors have the same gender.

6. (5 pts) Given the languages below, describe the new languages created using the simplest notation.

$$\Sigma = \{a, b, c\}$$

$$L_1 = \{b^n \mid n \geq 1\}$$

$$L_2 = \{ba^n \mid n \geq 0\}$$

$$L_3 = \{b^n a^n \mid n > 0\}$$

(a) $L_1 \cap \Sigma^* =$

(b) $L_2 \cap L_3 =$

(c) $L_1 \circ L_1 =$

(d) $L_2 \circ L_2 =$

(e) $L_2 \circ L_2^R =$

7. (12 pts) Consider each of the following languages. Use JFLAP to check your answer.

(a) $\Sigma = \{a, b\}$, $L = \{b, ab, bab\}$

i. write a grammar that generates the language.

(b) $\Sigma = \{a, b\}$, $L = \{a^n b^m \mid n > 0, m \geq 0\}$

i. list 3 strings in the language

ii. list 3 strings formed from the alphabet that are not in the language

iii. write a grammar that generates the language.

(c) $\Sigma = \{a, b\}$, $L = \{a^{2n} b^n \mid n > 0\}$

i. list 3 strings in the language

ii. list 3 strings formed from the alphabet that are not in the language

iii. write a grammar that generates the language.

(d) $\Sigma = \{a, b, c\}$, $L = \{a^m b^n c^m \mid n > 0, m \geq 0\}$

i. list 3 strings in the language

ii. list 3 strings formed from the alphabet that are not in the language

iii. write a grammar that generates the language.

(e) $\Sigma = \{a, b\}$, $L = \{w \in \Sigma^* \mid n_a(w) = 2\}$, ($n_a(w)$ means number of a 's in w)

i. list 3 strings in the language

ii. list 3 strings formed from the alphabet that are not in the language

iii. write a grammar that generates the language.