Announcements:

- This is a math course with systems applications. Prereq: CompSci 201, Compsci 230 or equiv.
- Course web page:

  www.cs.duke.edu/courses/

  spring26/compsci334

  Familiarize yourself with all parts of the web page.
- Flipped class - reading/quizzes BEFORE
- Read Chapter 1 in the Linz/Rodger book for next time.
- Complete the reading quizzes on Canvas before class.

  (Due to Drop/add, QZ01-QZ05 turn off Jan. 22 10, 11:45am!!)
- Course bulletin board: Ed Discussion (get to from Canvas)

- **Course participation required!**

What will we do in Compsci 334?

Questions

- **Can you write a program to determine if a string is an integer?**

  9998.89 8abab 789342

  yes!

- **Can you do this if your machine had no additional memory other than the program? (can't store any values and look at them again)**

  yes!

- **Can you write a program to determine if the following are correct arithmetic expressions?**

$$((34 + 7 * (18/6)))$$

$$(((((((a + b) + c) * d(e + f)))))$$

Yes!

- **Can you do this if your machine had no additional memory other than the program?**

No, need memory
need a stack
or something

- **Can you write a program to determine the value of the following expression?**

$$((34 + 7 * (18/6)))$$

yes

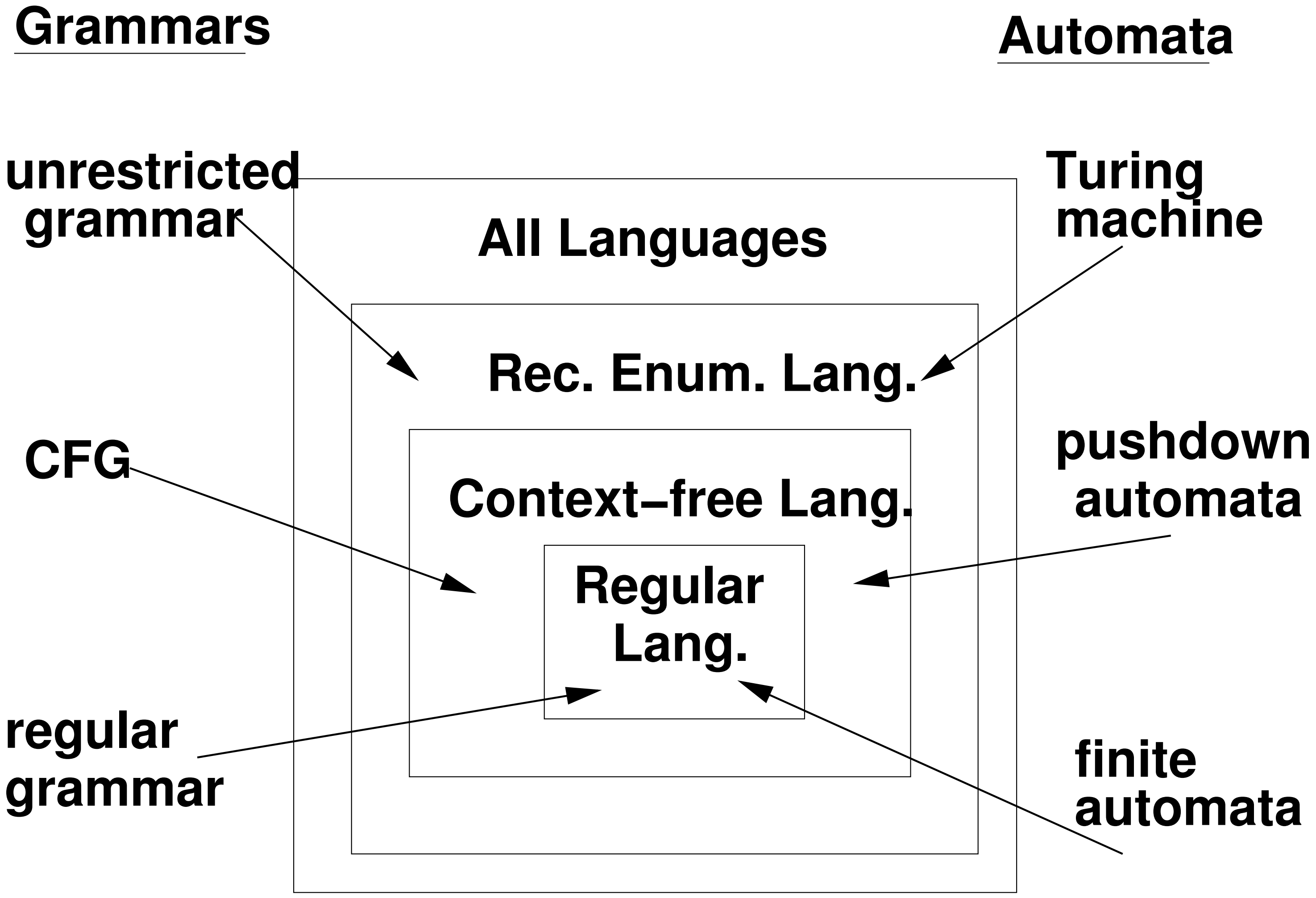
- **Can you write a program to determine if a file is a valid Java program?**

yes, syntax

- **Can you write a program to determine if a Java program given as input will ever halt?**

No! hard problem

Language Hierarchy



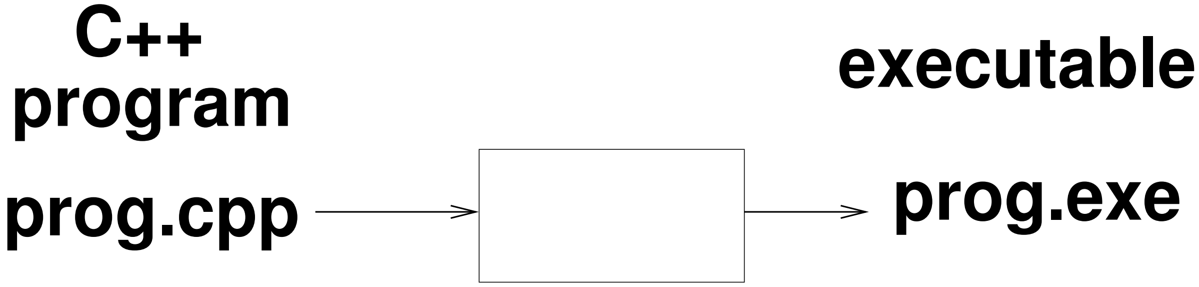
Power of Machines

automata	Can do?	Can't do?
FA **(no memory)**	recognize integers	recognize arithmetic expression
PDA **(stack)**	arithmetic expressions	evaluate expr
TM **(infinite)**	compute expr	decide if halts

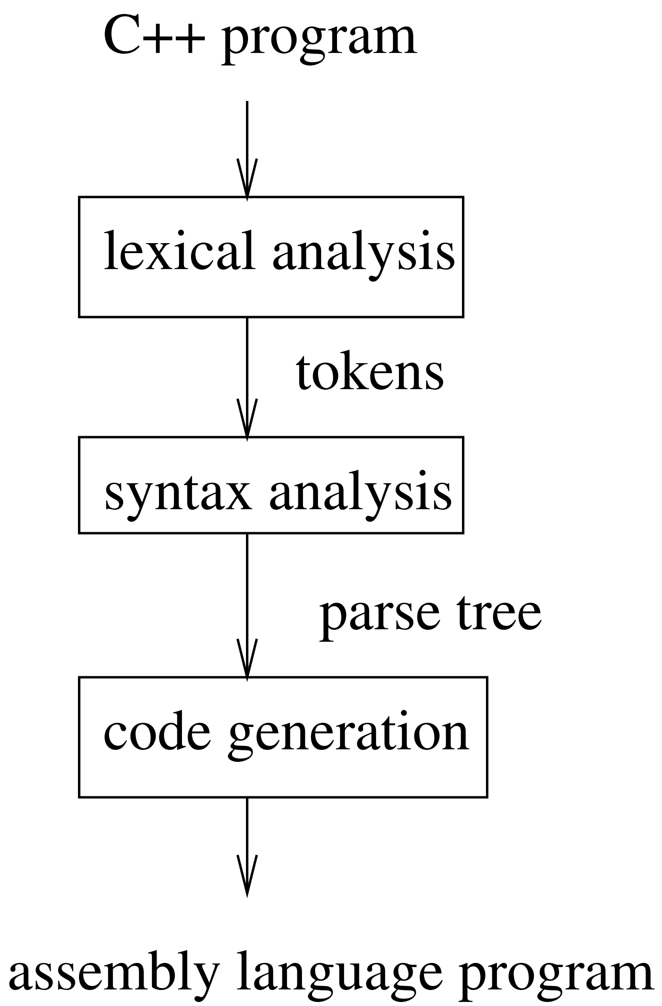
Application

Compiler

- **Our focus - Question: Given a program in some language (say Java or C++) - is it valid?**
- **Question: language L, program P - is P valid?**
- **Other things to consider, how is the compilation process different for different programming languages? (Java vs C++?)**



Stages of a Compiler



C++ program

lexical analysis

tokens

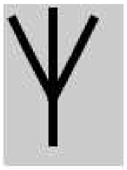
syntax analysis

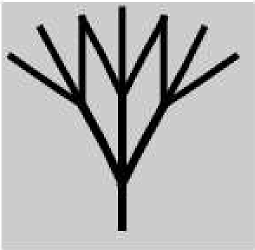
parse tree

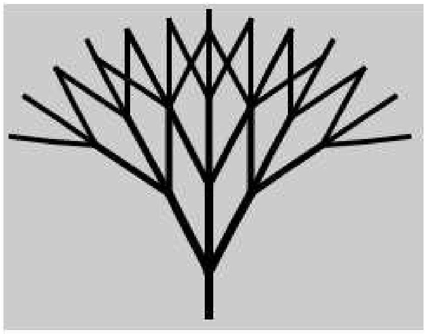
code generation

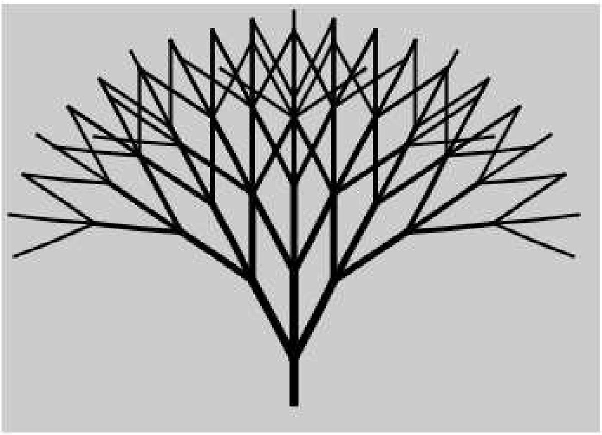
assembly language program

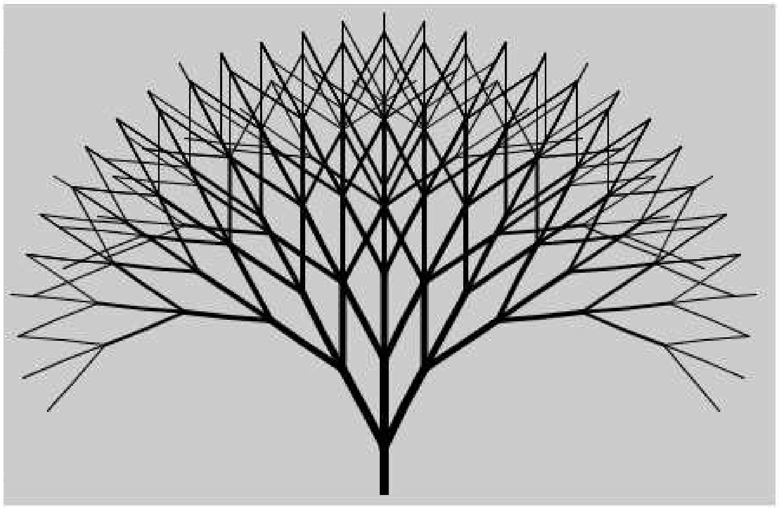
L-Systems - Model the Growth of Plants











Chapter 1 - Set Theory

A Set is a collection of elements.

A={1,4,6,8}, B={2,4,8},

C={3,6,9,12,...}, D={4,8,12,16,...}

- **(union) A∪B=** {1,2,4,6,8}
- **(intersection) A∩B=** {4,8}
- **C∩D=** {12, 24, ...}
- **(member of) 42 ∈ C?** yes
- **(subset) B⊂C?** No 2+4 not in
- **B∩A ⊆D?** yes
- **|B|=** 3
- **(product) A×B=** {(1,2),(1,4),(1,8),(4,2),(4,4),(4,8),(6,2),(6,4),(6,8),(8,2)(8,4)(8,8)
- **|A×B|=** 12
- **∅ ∈ B∩C?** NO
- **(powerset)** 2^B**=** {∅,{2},{4}{8}, {2,4},{2,8},{4,8},{2,4,8

Example What are all the subsets of $\{3, 5\}$?

$\emptyset$ $\{3\}$ $\{5\}$ $\{3,5\}$

How many subsets does a set S have?

$\|S\|$	**number of subsets**
0	1
1	2
2	4
3	8
4	16

n

$n+1$

How do you prove? Set S has $2^{|S|}$ subsets.

Technique: Proof by Induction

1. **Basis: P(1)?** smallest instance
2. **I.H.**

   Assume P(n) is true for 1,2,...,n
3. **I.S.**

   Show P(n+1) is true (using I.H.)

Set S has $2^{|S|}$ subsets.

Proof:

1. Basis: $|S|=0$ has one element. $2^0=1$ ✓

2. I.H. Assume $2^{|S|}$ is equal to the number of subsets in S for all $|S| \le n$

3. I.S. Show for $|S|=n+1$ that there are 2^{n+1} subsets

Take one element out $T \cup \{a\} = S$

There are 2^n subsets in T

S has all subsets of T plus a copy of each with a added.

OR $2 *$ number of subsets in T

$$2 * 2^n = 2^{n+1}$$

✓

Ch. 1: 3 Major Concepts

- **languages**
- **grammars**
- **automata**

Languages

- **Σ - set of symbols, alphabet**
- **string - finite sequence of symbols**
- **language - set of strings defined over Σ**

alphabet Σ

Examples

- $\Sigma = \{0,1,2,3,4,5,6,7,8,9\}$ $00 \notin L$

  L$=\{0, 1, 2, ..., 12, 13, 14, ...\}$
- Σ=$\{a, b, c\}$

  L$=\{ab, ac, cabb\}$
- Σ=$\{a, b\}$

  L$=\{a^n b^n \mid n > 0\}$ $= \{ab, aabb, aaabbb, \ldots\}$

Notation

- **symbols in alphabet: a, b, c, d, ...**
- **string names: u,v,w,...**

Definition of concatenation

Let w=$a_1 a_2 \ldots a_n$ **and v=**$b_1 b_2 \ldots b_m$

Then $w \circ v$ **OR wv=** $a_1 a_2 \ldots a_n b_1 b_2 \ldots b_m$

String Operations

strings: w=abbc, v=ab, u=c

- **size of string**

  $|w| + |v| = 6$

- **concatenation**

  $v^3 = \mathbf{vvv} = \mathbf{v{\circ}v{\circ}v} =$ ababab

- $v^0 = \lambda$ empty string
- $w^R =$ cbba
- $|vv^Rw|=$ 8
- $\mathbf{ab} \circ \lambda=$ ab

Definition

Σ^* concatenate 0 or more

Example

$\Sigma = \{a, b\}$

$\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aaa, \ldots\}$

$\Sigma^+ = \Sigma^* - \{\lambda\}$

Examples

$\Sigma = \{a, b, c\}$, $L_1 = \{ab, bc, aba\}$,
$L_2 = \{c, bc, bcc\}$

- $L_1 \cup L_2 = \{ab, bc, aba, bcc\}$
- $L_1 \cap L_2 = \{bc\}$
- $\overline{L_1} = \Sigma^* - L_1$
- $\overline{L_1 \cap L_2} = \Sigma^* - \{bc\}$
- $L_1 \circ L_2 = \{xy \mid x \in L_1$ **and** $y \in L_2\} =$

$\{abc, abbc, abbcc, bcc, bcbc, bcbcc, abac,$
$ababc, ababcc\}$

Definition

$L^0 = \{\lambda\}$

$L^2 = L \circ L$

$L^3 = L \circ L \circ L$

$L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \dots$

$L^+ = L^1 \cup L^2 \cup L^3 \dots$

Grammars

Grammar for english

<sentence> → <subject><verb><d.o.>

<subject> → <noun> | <article><noun>

<verb> → hit | ran | ate

<d.o.> → <article><noun> | <noun>

<noun> → Fritz | ball

<article> → the | an | a

Examples (derive a sentence)

Fritz hit the ball.

```
<sentence> -> <subject><verb><d.o>
           -> <noun><verb><d.o>
           -> Fritz <verb><d.o.>
           -> Fritz hit <d.o.>
           -> Fritz hit <article><noun>
           -> Fritz hit the <noun>
           -> Fritz hit the ball
```

Can we also derive the sentences?

The ball hit Fritz. yes

The ball ate the ball yes

Syntactically correct? yes

Semantically correct? no ?

Grammar

G=(V,T,S,P) where

- **V - variables (or nonterminals)** can be replaced
- **T - terminals** final, not replaced
- **S - start variable (S$\in$V)**
- **P - productions (rules)**
  x$\rightarrow$y x$\in$(V$\cup$ T)$^+$, y$\in$(V$\cup$T)*

at least one

Definition

w $\Rightarrow$ z   w derives z
w $\overset{*}{\Rightarrow}$ z   derives in 0 or more steps
w $\overset{+}{\Rightarrow}$ z   derives in 1 or more steps

Definition of Language of a grammar - L(G)

G=(V,T,S,P)

L(G)={w$\in$ T^* | S $\overset{*}{\Rightarrow}$ w}

Example

G=({S}, {a,b}, S, P)

P={S→ aaS, S→ b}

L(G)= $\{b, aab, aaaab, \ldots\ldots\}$
$= \{(aa)^n b \mid n \geq 0\}$ STOPPED

Example

L(G) $= \{a^n ccb^n \mid n > 0\}$

G =

Example

G=({S}, {a,b}, S, P)

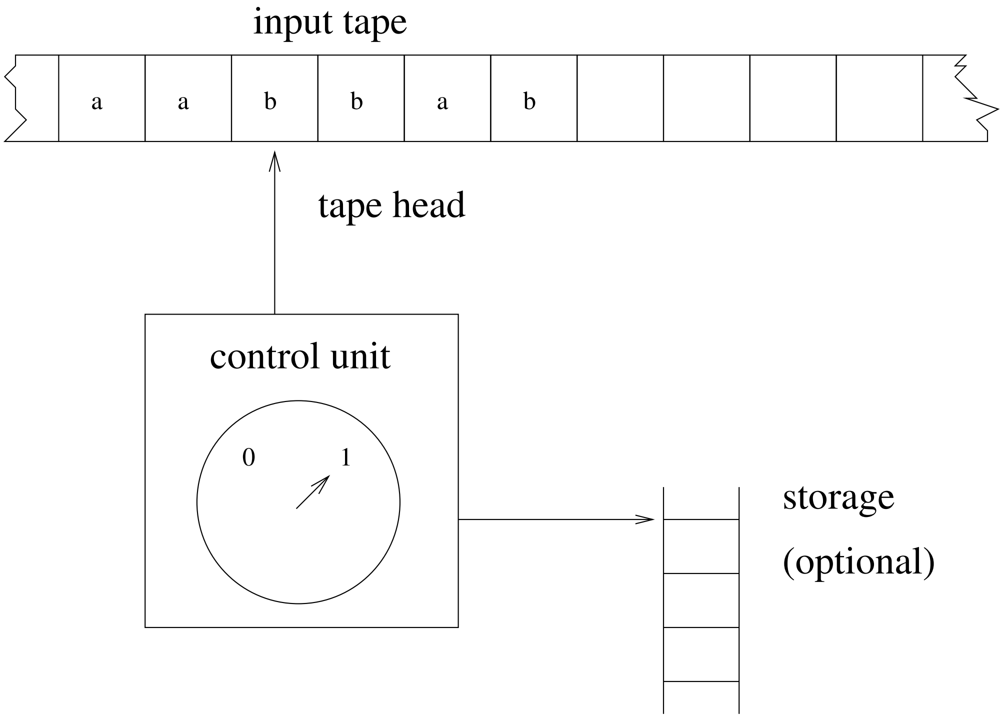
P={S→ aSb, S→ SS, S→ ab}

Which of these strings $aabb, abab, abba, babab$ **can be generated by this grammar? Show the derivations.**

L(G) =

Automata

Abstract model of a digital computer


input tape
a
a
b
b
a
b
tape head
control unit
0
1
storage
(optional)