

10 points

The problem below is to be done in groups of 2-3 students. The work is to be written up and turned in with all student names from the group on it.

1. The following game is played between a customer and a waitress. The customer places four glasses on a revolving tray, arranged in a circle. Each glass is either right-side-up or upside down. The waitress is blindfolded and wears boxing gloves so she cannot see or feel which way the glasses are placed, but the goal is to turn all the glasses right-side-up.

The game is played in rounds. The waitress can turn over 1-4 glasses. If the glasses are all right-side-up, the waitress is told that she has won. Otherwise, the customer rotates the tray either 90, 180, 270, or 360 degrees and the waitress tries again. The waitress is never told what the status of the current glasses is, unless they form a winning solution.

Is there a strategy with which the waitress eventually wins every time? That is, is there a sequence of glasses to turn over (maybe 2 adjacent glasses the first time, 1 glass the second time, ...) such that eventually the waitress is guaranteed to win?

If so, sketch out a solution on paper. If not, sketch out a proof why there is not a winning sequence.