

## Announcements:

- This is a math course with systems applications. Prereq: CompSci 201, CompSci 230 or equiv.
- Course web page:  
[www.cs.duke.edu/courses/spring26/compsci334](http://www.cs.duke.edu/courses/spring26/compsci334)  
Familiarize yourself with all parts of the web page.
- Flipped class - reading/quizzes BEFORE
- Read Chapter 1 in the Linz/Rodger book for next time.
- Complete the reading quizzes on Canvas before class.  
(Due to Drop/add, QZ01-QZ05 turn off Jan. 22 10, 11:45am!!)
- Course bulletin board: Ed Discussion (get to from Canvas)

- **Course participation required!**

# What will we do in Compsci 334?

## Questions

- Can you write a program to determine if a string is an integer?

9998.89

8abab

789342

- Can you do this if your machine had no additional memory other than the program? (can't store any values and look at them again)

- Can you write a program to determine if the following are correct arithmetic expressions?

$$((34 + 7 * (18/6)))$$

$$(((((((a + b) + c) * d(e + f))))))$$

- Can you do this if your machine had no additional memory other than the program?

- Can you write a program to determine the value of the following expression?

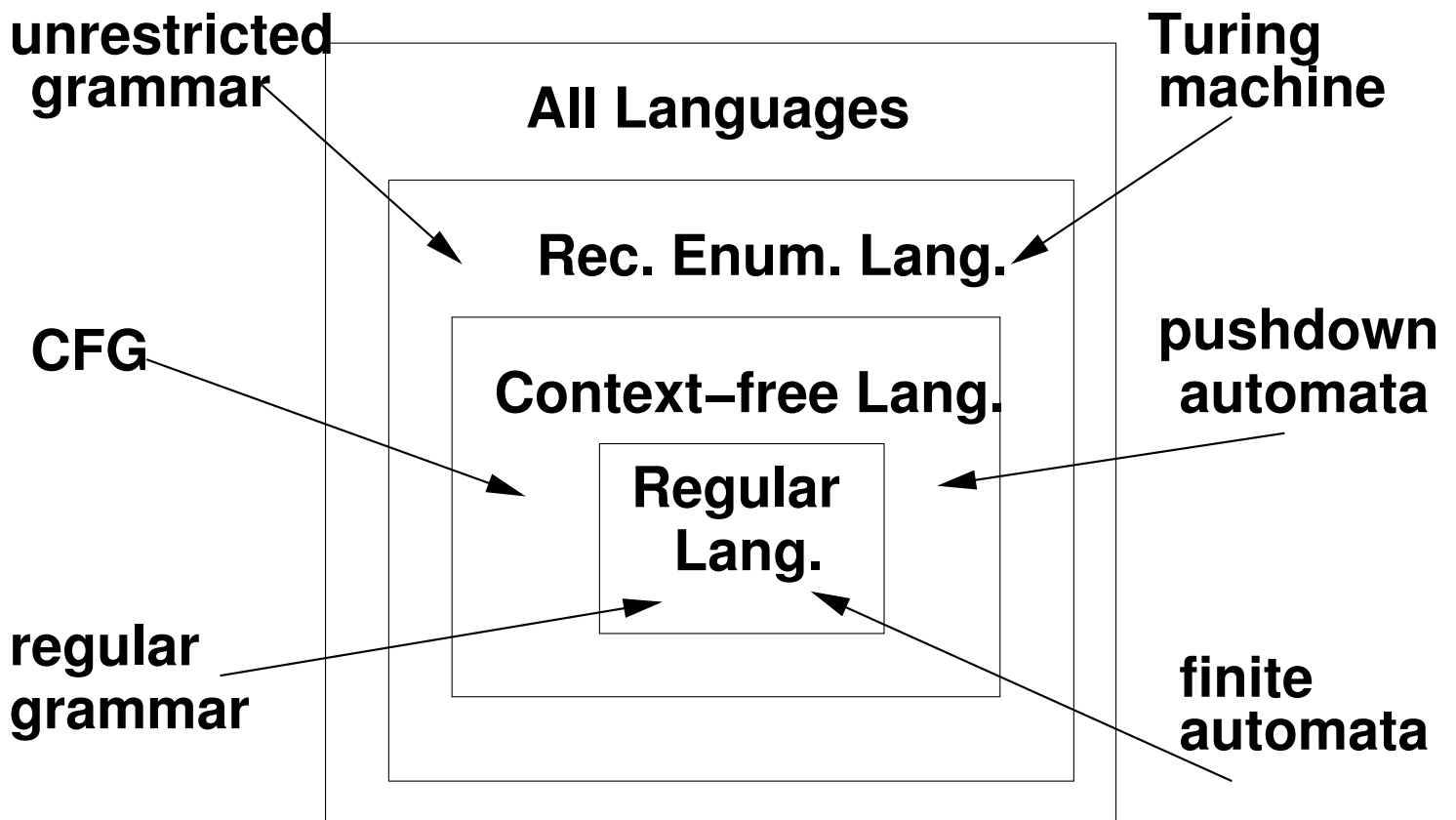
$$((34 + 7 * (18/6)))$$

- Can you write a program to determine if a file is a valid Java program?
- Can you write a program to determine if a Java program given as input will ever halt?

# Language Hierarchy

## Grammars

## Automata



# Power of Machines

automata

Can do?

Can't do?

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FA

(no memory)

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PDA

(stack)

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TM

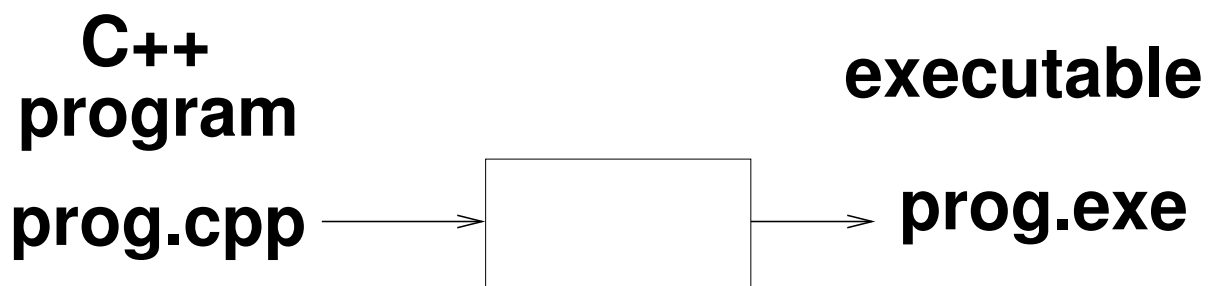
(infinite)

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# Application

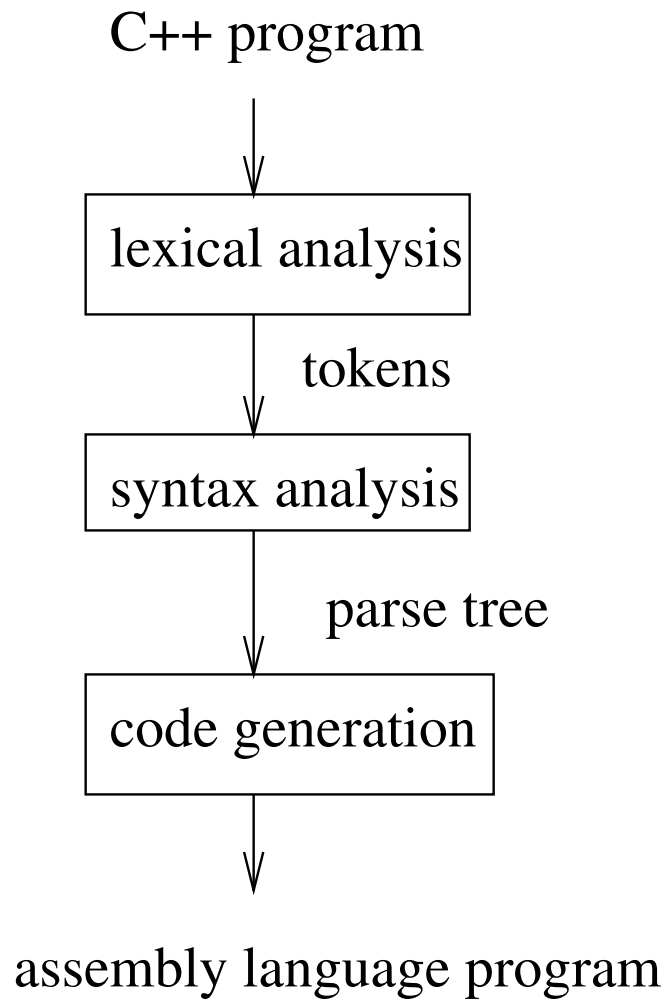
## Compiler

- Our focus - Question: Given a program in some language (say Java or C++) - is it valid?
- Question: language L, program P - is P valid?
- Other things to consider, how is the compilation process different for different programming languages? (Java vs C++?)

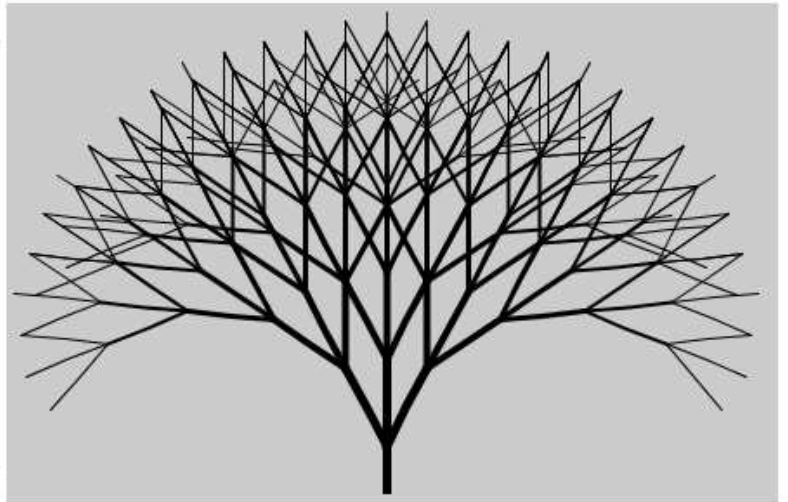
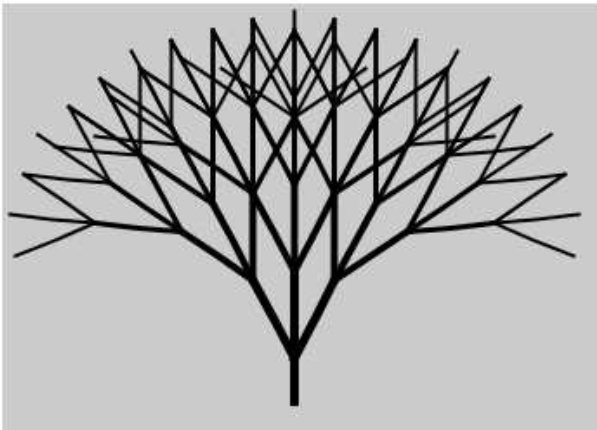
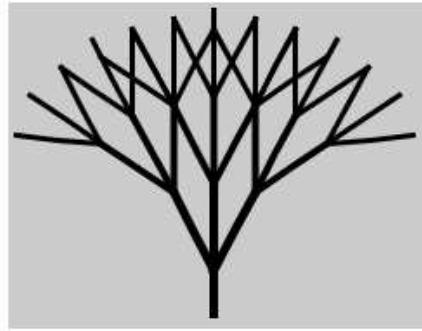




# Stages of a Compiler



# L-Systems - Model the Growth of Plants



## Chapter 1 - Set Theory

A Set is a collection of elements.

$$A = \{1, 4, 6, 8\}, B = \{2, 4, 8\},$$

$$C = \{3, 6, 9, 12, \dots\}, D = \{4, 8, 12, 16, \dots\}$$

- (union)  $A \cup B =$
- (intersection)  $A \cap B =$
- $C \cap D =$
- (member of)  $42 \in C?$
- (subset)  $B \subset C?$
- $B \cap A \subseteq D?$
- $|B| =$
- (product)  $A \times B =$
- $|A \times B| =$
- $\emptyset \in B \cap C?$
- (powerset)  $2^B =$

**Example** What are all the subsets of  $\{3, 5\}$ ?

**How many subsets does a set  $S$  have?**

$ S $	number of subsets
-------	-------------------

0	
---	--

1	
---	--

2	
---	--

3	
---	--

4	
---	--

How do you prove? Set  $S$  has  $2^{|S|}$  subsets.

Technique: Proof by Induction

1. Basis:  $P(1)$ ?

2. I.H.

Assume  $P(n)$  is true for  $1, 2, \dots, n$

3. I.S.

Show  $P(n+1)$  is true (using I.H.)

Set  $S$  has  $2^{|S|}$  subsets.

Proof:

1. Basis:

2. I.H. Assume

3. I.S. Show

## Ch. 1: 3 Major Concepts

- languages
- grammars
- automata

# Languages

- $\Sigma$  - set of symbols, alphabet
- string - finite sequence of symbols
- language - set of strings defined over  $\Sigma$

alphabet  $\Sigma$

## Examples

- $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$   
 $L = \{0, 1, 2, \dots, 12, 13, 14, \dots\}$
- $\Sigma = \{a, b, c\}$   
 $L = \{ab, ac, cabb\}$
- $\Sigma = \{a, b\}$   
 $L = \{a^n b^n \mid n > 0\}$



## Notation

- symbols in alphabet: a, b, c, d, ...
- string names: u, v, w, ...

## Definition of concatenation

Let  $w = a_1 a_2 \dots a_n$  and  $v = b_1 b_2 \dots b_m$

Then  $w \circ v$  OR  $wv =$

# String Operations

strings:  $w=abbc$ ,  $v=ab$ ,  $u=c$

- size of string

$$|w| + |v| =$$

- concatenation

$$v^3 = \mathbf{vvv} = \mathbf{v \circ v \circ v} =$$

- $v^0 =$

- $w^R =$

- $|vv^Rw| =$

- $\mathbf{ab} \circ \lambda =$

## Definition

$\Sigma^*$  concatenate 0 or more

## Example

$$\Sigma = \{a, b\}$$

$$\Sigma^* =$$

$$\Sigma^+ =$$

## Examples

$$\Sigma = \{a, b, c\}, L_1 = \{ab, bc, aba\},$$
$$L_2 = \{c, bc, bcc\}$$

$$\bullet L_1 \cup L_2 =$$

$$\bullet L_1 \cap L_2 =$$

$$\bullet \overline{L_1} =$$

$$\bullet \overline{L_1 \cap L_2} =$$

$$\bullet L_1 \circ L_2 = \{xy \mid x \in L_1 \text{ and } y \in L_2\} =$$

## Definition

$$L^0 = \{\lambda\}$$

$$L^2 = L \circ L$$

$$L^3 = L \circ L \circ L$$

$$L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \dots$$

$$L^+ = L^1 \cup L^2 \cup L^3 \dots$$

# Grammars

## Grammar for english

$\langle \text{sentence} \rangle \rightarrow$

$\langle \text{subject} \rangle \langle \text{verb} \rangle \langle \text{d.o.} \rangle$

$\langle \text{subject} \rangle \rightarrow \langle \text{noun} \rangle \mid$

$\langle \text{article} \rangle \langle \text{noun} \rangle$

$\langle \text{verb} \rangle \rightarrow \text{hit} \mid \text{ran} \mid \text{ate}$

$\langle \text{d.o.} \rangle \rightarrow \langle \text{article} \rangle \langle \text{noun} \rangle \mid \langle \text{noun} \rangle$

$\langle \text{noun} \rangle \rightarrow \text{Fritz} \mid \text{ball}$

$\langle \text{article} \rangle \rightarrow \text{the} \mid \text{an} \mid \text{a}$

## Examples (derive a sentence)

Fritz hit the ball.

```
<sentence> -> <subject><verb><d.o>
            -> <noun><verb><d.o>
            -> Fritz <verb><d.o.>
            -> Fritz hit <d.o.>
            -> Fritz hit <article><noun>
            -> Fritz hit the <noun>
            -> Fritz hit the ball
```

Can we also derive the sentences?

The ball hit Fritz.

The ball ate the ball

Syntactically correct?

Semantically correct?

## Grammar

$G=(V,T,S,P)$  where

- $V$  - variables (or nonterminals)
- $T$  - terminals
- $S$  - start variable ( $S \in V$ )
- $P$  - productions (rules)  
 $x \rightarrow y \quad x \in (V \cup T)^+, y \in (V \cup T)^*$

## Definition

$w \Rightarrow z$   $w$  derives  $z$

$w \xRightarrow{*} z$   $w$  derives in 0 or more steps

$w \xRightarrow{+} z$   $w$  derives in 1 or more steps

Definition of Language of a grammar -  
 $L(G)$

$G=(V,T,S,P)$

$L(G)=\{w \in T^* \mid S \xRightarrow{*} w\}$



Example

$$G = (\{S\}, \{a, b\}, S, P)$$

$$P = \{S \rightarrow aaS, S \rightarrow b\}$$

$$L(G) =$$

Example

$$L(G) = \{a^n ccb^n \mid n > 0\}$$

$$G =$$

Example

$$G = (\{S\}, \{a, b\}, S, P)$$

$$P = \{S \rightarrow aSb, S \rightarrow SS, S \rightarrow ab\}$$

Which of these strings

*aabb, abab, abba, babab* can be generated by this grammar? Show the derivations.

$$L(G) =$$

# Automata

## Abstract model of a digital computer

