

## Announcements:

- This is a math course with systems applications. Prereq: CompSci 201, CompSci 230 or equiv.
- Course web page:  
[www.cs.duke.edu/courses/  
spring26/compsci334](http://www.cs.duke.edu/courses/spring26/compsci334)  
Familiarize yourself with all parts  
of the web page.
- Flipped class - reading/quizzes  
BEFORE
- Read Chapter 1 in the Linz/Rodger  
book for next time.
- Complete the reading quizzes on  
Canvas before class.  
(Due to Drop/add, QZ01-QZ05  
turn off Jan. 22 10, 11:45am!!)
- Course bulletin board: Ed  
Discussion (get to from Canvas)

- Course participation required!

# What will we do in CompSci 334?

## Questions

- Can you write a program to determine if a string is an integer?

9998.89      8abab      789342

- Can you do this if your machine had no additional memory other than the program? (can't store any values and look at them again)

- Can you write a program to determine if the following are correct arithmetic expressions?

$$((34 + 7 * (18/6)))$$

$$((((((a + b) + c) * d(e + f)))))$$

- Can you do this if your machine had no additional memory other than the program?

- Can you write a program to determine the value of the following expression?

$$((34 + 7 * (18/6)))$$

- Can you write a program to determine if a file is a valid Java program?
- Can you write a program to determine if a Java program given as input will ever halt?

# Language Hierarchy

## Grammars

**unrestricted grammar**

**CFG**

**regular grammar**

## Automata

**Turing machine**

**pushdown automata**

**finite automata**

**All Languages**

**Rec. Enum. Lang.**

**Context-free Lang.**

**Regular Lang.**

# Power of Machines

automata

Can do?

Can't do?

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FA

(no memory)

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PDA

(stack)

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TM

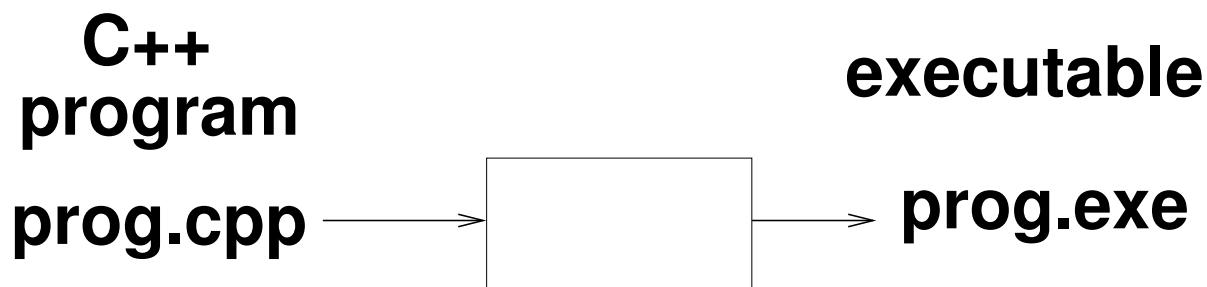
(infinite)

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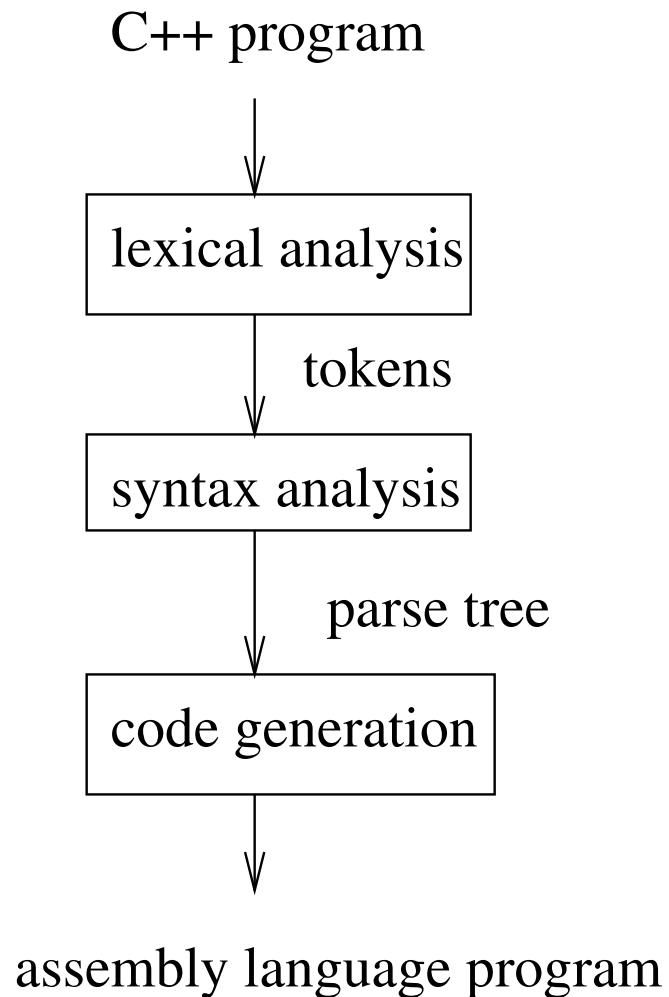
# Application

## Compiler

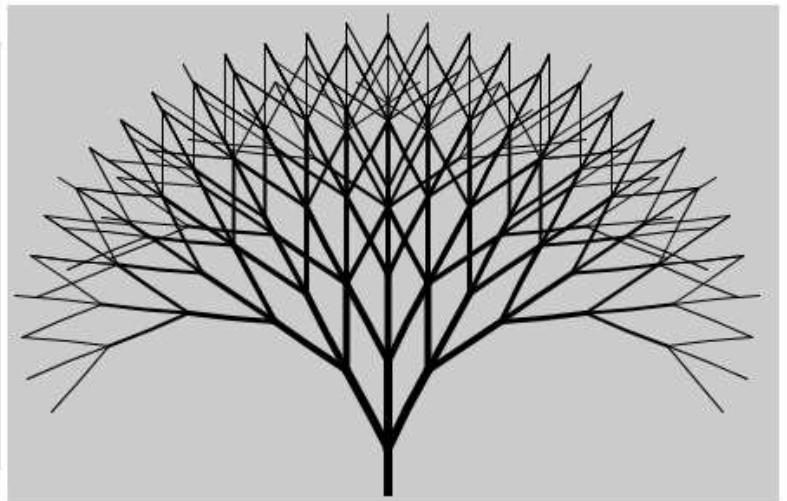
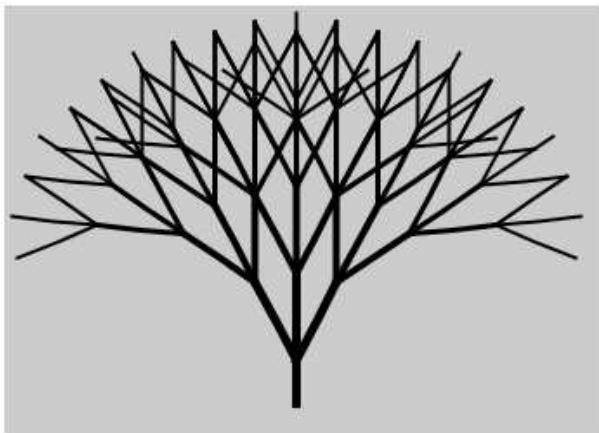
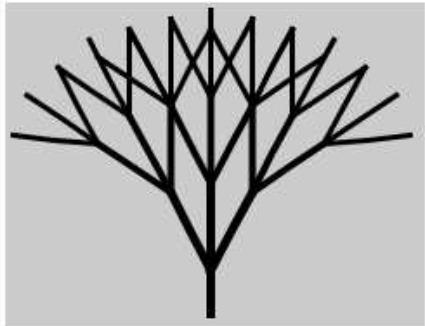
- Our focus - Question: Given a program in some language (say Java or C++) - is it valid?
- Question: language L, program P - is P valid?
- Other things to consider, how is the compilation process different for different programming languages? (Java vs C++?)



# Stages of a Compiler



# L-Systems - Model the Growth of Plants



# Chapter 1 - Set Theory

A Set is a collection of elements.

$$A = \{1, 4, 6, 8\}, B = \{2, 4, 8\},$$

$$C = \{3, 6, 9, 12, \dots\}, D = \{4, 8, 12, 16, \dots\}$$

- (union)  $A \cup B =$
- (intersection)  $A \cap B =$
- $C \cap D =$
- (member of)  $42 \in C?$
- (subset)  $B \subset C?$
- $B \cap A \subseteq D?$
- $|B| =$
- (product)  $A \times B =$
- $|A \times B| =$
- $\emptyset \in B \cap C?$
- (powerset)  $2^B =$

**Example** What are all the subsets of  $\{3, 5\}$ ?

How many subsets does a set  $S$  have?

$ S $	number of subsets
0	
1	
2	
3	
4	

How do you prove? Set  $S$  has  $2^{|S|}$  subsets.

Technique: Proof by Induction

1. Basis:  $P(1)$ ?

2. I.H.

Assume  $P(n)$  is true for  $1, 2, \dots, n$

3. I.S.

Show  $P(n+1)$  is true (using I.H.)

Set  $S$  has  $2^{|S|}$  subsets.

**Proof:**

**1. Basis:**

**2. I.H. Assume**

**3. I.S. Show**

# Ch. 1: 3 Major Concepts

- languages
- grammars
- automata

# Languages

- $\Sigma$  - set of symbols, alphabet
- string - finite sequence of symbols
- language - set of strings defined over  $\Sigma$

alphabet  $\Sigma$

## Examples

- $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$   
 $L = \{0, 1, 2, \dots, 12, 13, 14, \dots\}$
- $\Sigma = \{a, b, c\}$   
 $L = \{ab, ac, cabb\}$
- $\Sigma = \{a, b\}$   
 $L = \{a^n b^n \mid n > 0\}$

## Notation

- symbols in alphabet: a, b, c, d, ...
- string names: u, v, w, ...

## Definition of concatenation

Let  $w = a_1 a_2 \dots a_n$  and  $v = b_1 b_2 \dots b_m$

Then  $w \circ v$  OR  $wv =$

# String Operations

strings:  $w=abbc$ ,  $v=ab$ ,  $u=c$

- **size of string**

$$|w| + |v| =$$

- **concatenation**

$$v^3 = \mathbf{vvv} = \mathbf{v} \circ \mathbf{v} \circ \mathbf{v} =$$

- $v^0 =$

- $w^R =$

- $|vv^Rw| =$

- $\mathbf{ab} \circ \lambda =$

# Definition

$\Sigma^*$  concatenate 0 or more

## Example

$$\Sigma = \{a, b\}$$

$$\Sigma^* =$$

$$\Sigma^+ =$$

## Examples

$$\Sigma = \{a, b, c\}, L_1 = \{ab, bc, aba\},$$
$$L_2 = \{c, bc, bcc\}$$

- $L_1 \cup L_2 =$
- $L_1 \cap L_2 =$
- $\overline{L_1} =$
- $\overline{L_1 \cap L_2} =$
- $L_1 \circ L_2 = \{xy \mid x \in L_1 \text{ and } y \in L_2\} =$

## Definition

$$L^0 = \{\lambda\}$$

$$L^2 = L \circ L$$

$$L^3 = L \circ L \circ L$$

$$L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \dots$$

$$L^+ = L^1 \cup L^2 \cup L^3 \dots$$

# Grammars

## Grammar for english

**<sentence>** →

**<subject><verb><d.o.>**

**<subject>** → **<noun>** |

**<article><noun>**

**<verb>** → hit | ran | ate

**<d.o.>** → **<article><noun>** | **<noun>**

**<noun>** → Fritz | ball

**<article>** → the | an | a

## Examples (derive a sentence)

Fritz hit the ball.

```
<sentence> -> <subject><verb><d.o>
-> <noun><verb><d.o>
-> Fritz <verb><d.o.>
-> Fritz hit <d.o.>
-> Fritz hit <article><noun>
-> Fritz hit the <noun>
-> Fritz hit the ball
```

Can we also derive the sentences?

The ball hit Fritz.

The ball ate the ball

Syntactically correct?

Semantically correct?

# Grammar

$G = (V, T, S, P)$  where

- $V$  - variables (or nonterminals)
- $T$  - terminals
- $S$  - start variable ( $S \in V$ )
- $P$  - productions (rules)  
 $x \rightarrow y \quad x \in (V \cup T)^+, \quad y \in (V \cup T)^*$

## Definition

$w \Rightarrow z$   $w$  derives  $z$

$w \xrightarrow{*} z$  derives in 0 or more steps

$w \xrightarrow{+} z$  derives in 1 or more steps

**Definition of Language of a grammar -  $L(G)$**

$G = (V, T, S, P)$

$L(G) = \{w \in T^* \mid S \xrightarrow{*} w\}$

**Example**

$$G = (\{S\}, \{a, b\}, S, P)$$

$$P = \{S \rightarrow aaS, S \rightarrow b\}$$

$$L(G) =$$

**Example**

$$L(G) = \{a^n c c b^n \mid n > 0\}$$

$$G =$$

**Example**

$$G = (\{S\}, \{a, b\}, S, P)$$

$$P = \{S \rightarrow aSb, S \rightarrow SS, S \rightarrow ab\}$$

**Which of these strings**

*aabb, abab, abba, babab* can be generated by this grammar? Show the derivations.

$$L(G) =$$

# Automata

## Abstract model of a digital computer

