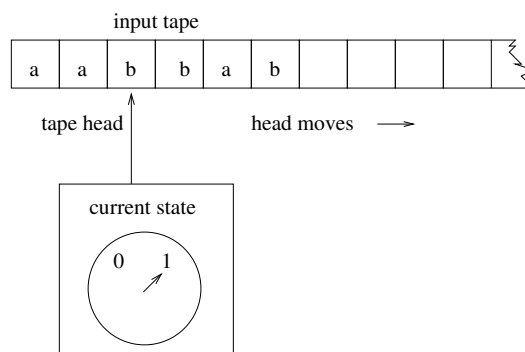


CompSci 334 - Mathematical Foundations of CS
 Dr. Susan Rodger
 Section: Finite Automata (Ch. 2) (handout)

Deterministic Finite Acceptor (or Automata)

A DFA = $(Q, \Sigma, \delta, q_0, F)$



where

Q is finite set of states

Σ is tape (input) alphabet

q_0 is initial state

$F \subseteq Q$ is set of final states.

$\delta: Q \times \Sigma \rightarrow Q$

Example: Create a DFA that accepts even binary numbers.

Transition Diagram:

$M = (Q, \Sigma, \delta, q_0, F) =$

Tabular Format

	0	1
q0		
q1		

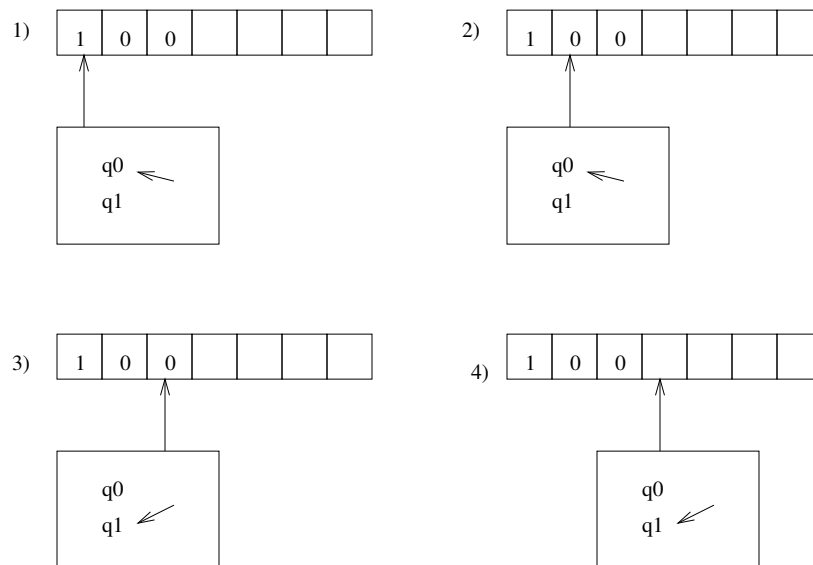
Example of a move: $\delta(q_0, 1) =$

Algorithm for DFA:

Start in start state with input on tape
q = current state
s = current symbol on tape
while (s != blank) do
 q = $\delta(q,s)$
 s = next symbol to the right on tape
if $q \in F$ then accept

Example of a trace: 11010

Pictorial Example of a trace for 100:



Definition:

$$\delta^*(q, \lambda) = q$$

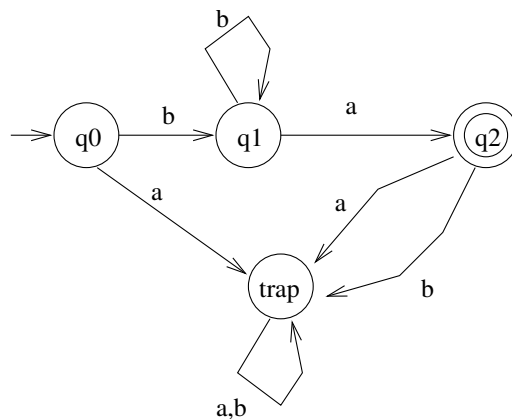
$$\delta^*(q, wa) = \delta(\delta^*(q, w), a)$$

Definition The language accepted by a DFA $M=(Q,\Sigma,\delta,q_0,F)$ is set of all strings on Σ accepted by M. Formally,

$$L(M)=\{w \in \Sigma^* \mid \delta^*(q_0, w) \in F\}$$

Trap State

Example: $L(M) =$



You don't need to show trap states! Any arc not shown will by default go to a trap state.

Example:

$L = \{w \in \Sigma^* \mid w \text{ has an even number of a's and an even number of b's}\}$

Example: Create a DFA that accepts even binary numbers that have an even number of 1's.

Definition A language L is regular iff there exists DFA M s.t. $L=L(M)$.

Chapter 2.2

Nondeterministic Finite Automata (or Acceptor)

Definition

An NFA= $(Q, \Sigma, \delta, q_0, F)$

where

Q is finite set of states

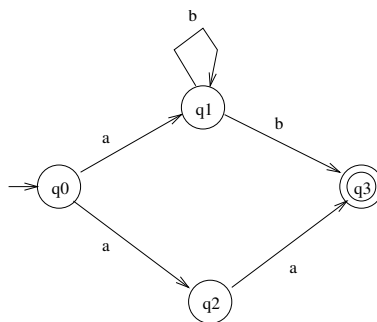
Σ is tape (input) alphabet

q_0 is initial state

$F \subseteq Q$ is set of final states.

$\delta: Q \times (\Sigma \cup \{\lambda\}) \rightarrow 2^Q$

Example



Note: In this example $\delta(q_0, a) =$

$L =$

Example

$L = \{(ab)^n \mid n > 0\} \cup \{a^n b \mid n > 0\}$

Definition $q_j \in \delta^*(q_i, w)$ if and only if there is a walk from q_i to q_j labeled w .

Example From previous example:

$\delta^*(q_0, ab) =$

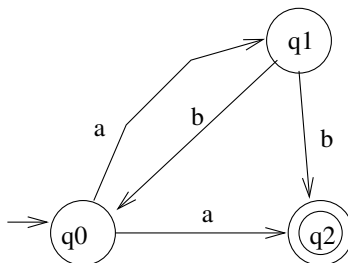
$\delta^*(q_0, aba) =$

Definition: For an NFA M , $L(M) = \{w \in \Sigma^* \mid \delta^*(q_0, w) \cap F \neq \emptyset\}$

The language accepted by nfa M is all strings w such that there exists a walk labeled w from the start state to final state.

2.3 NFA vs. DFA: Which is more powerful?

Example:



Theorem Given an NFA $M_N = (Q_N, \Sigma, \delta_N, q_0, F_N)$, then there exists a DFA $M_D = (Q_D, \Sigma, \delta_D, q_0, F_D)$ such that $L(M_N) = L(M_D)$.

Proof:

We need to define M_D based on M_N .

$Q_D =$

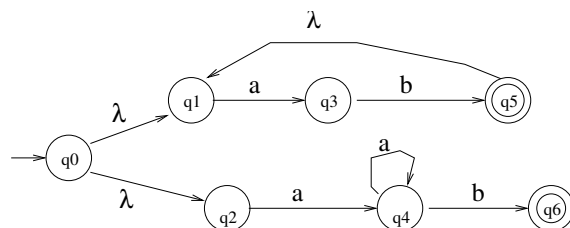
$F_D =$

$\delta_D :$

Algorithm to construct M_D

1. start state is $\{q_0\} \cup \text{closure}(q_0)$
2. While can add an edge
 - (a) Choose a state $A = \{q_i, q_j, \dots, q_k\}$ with missing edge for $a \in \Sigma$
 - (b) Compute $B = \delta^*(q_i, a) \cup \delta^*(q_j, a) \cup \dots \cup \delta^*(q_k, a)$
 - (c) Add state B if it doesn't exist
 - (d) add edge from A to B with label a
3. Identify final states
4. if $\lambda \in L(M_N)$ then make the start state final.

Example:



Properties and Proving - Problem 1

Consider the property Replace_one_a_with_b or R1awb for short. If L is a regular, prove $R1awb(L)$ is regular.

The property R1awb applied to a language L replaces one a in each string with a b . If a string does not have an a , then the string is not in $R1awb(L)$.

Example 1: Consider $L = \{aaab, bbaa\}$

$R1awb(L) =$

Example 2: Consider $\Sigma = \{a, b\}$, $L = \{w \in \Sigma^* \mid w \text{ has an even number of } a\text{'s and an even number of } b\text{'s}\}$

$R1awb(L) =$

Proof:

Properties and Proving - Problem 2

Consider the property Truncate_all_preceding_b's or TruncPreb for short. If L is a regular, prove $\text{TruncPreb}(L)$ is regular.

The property TruncPreb applied to a language L removes all preceding b's in each string. If a string does not have an preceding b, then the string is the same in $\text{TruncPreb}(L)$.

Example 1: Consider $L = \{aaab, bbaa\}$

$\text{TruncPreb}(L) =$

Example 2: Consider $L = \{(bba)^n \mid n > 0\}$

$\text{TruncPreb}(L) =$

Proof:

Minimizing Number of states in DFA

Why?

Algorithm

- Identify states that are indistinguishable

These states form a new state

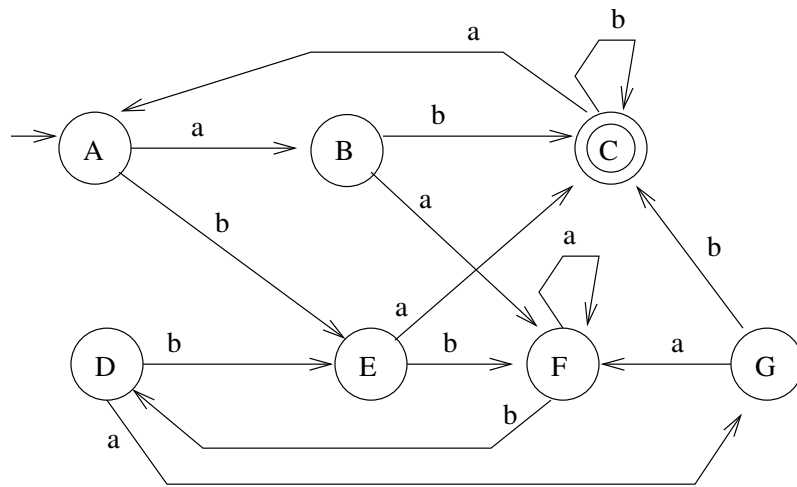
Definition Two states p and q are indistinguishable if for all $w \in \Sigma^*$

$$\begin{aligned}\delta^*(q, w) \in F &\Rightarrow \delta^*(p, w) \in F \\ \delta^*(p, w) \notin F &\Rightarrow \delta^*(q, w) \notin F\end{aligned}$$

Definition Two states p and q are distinguishable if $\exists w \in \Sigma^*$ s.t.

$$\begin{aligned}\delta^*(q, w) \in F &\Rightarrow \delta^*(p, w) \notin F \text{ OR} \\ \delta^*(q, w) \notin F &\Rightarrow \delta^*(p, w) \in F\end{aligned}$$

Example:



Example:

